Consumer Choice, Market Power, and Inflation*

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Abstract

We introduce consumer choice into a search-theoretic model of monetary exchange. In contrast to standard search models featuring bilateral meetings, consumers can meet multiple sellers and *choose* a seller with whom to trade. Market power is endogenized through competitive search and it is influenced by the degree of consumer choice. We consider the effects of greater consumer choice on both market power and the welfare cost of inflation. Surprisingly, we find that greater consumer choice can have a *non-monotonic* effect on market power. At lower levels of consumer choice, an increase in the degree of consumer choice tends to *increase* firms' market power, while the opposite is true at higher levels. When we calibrate the model to U.S. data, we find that despite greater consumer choice having a positive effect on welfare overall, it can also amplify the negative welfare effects of inflation, making it significantly more costly.

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1 Introduction

Consumer choice is an important feature of monetary exchange. When consumers purchase retail goods, they typically choose from a number of goods that are available simultaneously from a range of competing sellers. Choice is often idiosyncratic: different consumers might make different choices when faced with the same range of goods, and the same consumer might make different choices at different points in time. Discrete choice models with random utility shocks have been used extensively to study these kinds of choices in the large literature following Anderson, De Palma, and Thisse (1992), but these models do not feature monetary exchange.

Search-theoretic models have become the standard way of modelling the microfoundations of monetary exchange, as surveyed in Lagos, Rocheteau, and Wright (2017). However, meetings are typically one-on-one (or bilateral) in these models: each buyer meets at most one seller during a single period of time and can choose either to trade with that seller or wait. While many papers have incorporated random utility shocks or match-specific preference shocks into the process of monetary exchange, such as Lagos and Rocheteau (2005), these shocks influence only the quantities traded and the payments – not the choice of seller – because meetings are bilateral. In this way, there is no genuine role for what we call *consumer choice*, i.e. buyers' choice of seller.

This paper introduces consumer choice into a search-theoretic model of monetary exchange based on Rocheteau and Wright (2005). To show that consumer choice matters, we ask two main questions: How does a greater degree of consumer choice influence market power? How does a greater degree of consumer choice affect the welfare cost of inflation? To answer these questions, we use the theoretical framework developed in Mangin (2025), which introduces consumer choice into a search-theoretic model of retail trade with non-linear pricing and competitive search.

We extend the model in Mangin (2025) in two important ways. First, we introduce monetary exchange. Second, we incorporate a parameter α that represents the *degree of consumer choice*. This allows us to vary the extent to which consumers can make choices among different sellers about which goods to purchase. In turn, this flexibility enables us to answer two precise questions: Is the market power of firms higher or lower with greater consumer choice? Is the welfare cost of inflation higher or lower with greater consumer choice? The first question is interesting in its own right because we might expect firms to have greater market power when there is greater consumer choice, but we find that – surprisingly – this is not always the case. The second question is useful because our answer shows that modelling consumer choice is important for estimating the welfare cost of inflation – despite being neglected by standard search-theoretic models of monetary exchange with bilateral meetings.

First, we find that a greater degree of consumer choice α can affect market power in a *non-monotonic* manner. We define firms' market power as the *surplus share* of firms, which is endogenous. When we increase consumers' degree of choice, firms' market power is at first increasing in α at lower levels of consumer choice and then decreasing in α at higher levels. The same non-monotonic effect of consumer choice on firms' market power can be observed if we define firms' market power instead as the average markup over marginal cost. This non-monotonic effect of consumer choice on market power is surprising and counter-intuitive. We might expect greater consumer choice would always lead to a decrease in any measure of firms' market power.

Second, we find that a greater degree of consumer choice α significantly increases the welfare cost of inflation. As a result, economies in which buyers are more likely to have greater choice – for example, as a result of rising internet availability – may be more sensitive to the negative effects of inflation. This means that the same inflation rate may be more costly today than in earlier decades.

We model monetary exchange as in Rocheteau and Wright (2005), which is based on the Lagos and Wright (2005) alternating markets structure and also features endogenous seller entry. We focus on *competitive search equilibrium.* Buyers and sellers choose to enter submarkets in which terms of trade, or contracts, are posted by market makers. Within each submarket, buyers and sellers commit to trading at the posted terms of trade in that submarket and search frictions govern how buyers and sellers meet. Directed or competitive search is a natural alternative to bargaining in our environment because buyers can meet multiple sellers within a single meeting. It is also a natural benchmark for welfare analysis because directed or competitive search is often used to decentralize the constrained efficient allocation in search-theoretic environments, as discussed in Wright, Kircher, Julien, and Guerrieri (2021).

We model search frictions within submarkets by allowing two possibilities: buyer can meet sellers either through random, bilateral meetings or through *one-to-many meetings*. With probability $\alpha \in (0, 1]$, there are one-to-many meetings in which buyers meet a finite number of sellers (either zero, one, or more than one seller). In this case, we say that buyers have *consumer choice* because they can meet a number of sellers and choose the seller with whom they wish to trade. With probability $1 - \alpha$, there are random, bilateral meetings and there is no consumer choice. Buyers either meet one seller or no sellers. The standard approach in monetary search models of random, bilateral meetings within submarkets is therefore nested as a special limiting case where $\alpha \to 0$. This allows us to isolate the precise effect of consumer choice.

After a meeting takes place, nature draws an i.i.d. preference or utility shock specific to each seller in the meeting. The buyer then chooses to purchase from the seller that maximizes their net utility. If the buyer meets more than one seller, they can choose from among all the sellers they meet, while if the buyer meets only one seller, there is no choice and they can either trade or wait. Prior to trade, sellers cannot observe buyers' utility shocks; they are private information for the buyer. It is sometimes convenient to refer to the utility shock associated with a good as its *quality*, although this is an idiosyncratic perception of quality that is specific to a buyer.

With consumer choice, the relevant distribution of private buyer "types" is not the exogenous distribution of utility shocks but the *endogenous* distribution of utility shocks of *chosen* goods. This is because sellers know that if a buyer wants to trade, it means they must have first been chosen from among the other sellers the buyer has met. This *distribution of chosen goods* depends on three things: the seller-buyer ratio; the degree of consumer choice; and the exogenous distribution of utility shocks. With probability α, there is the possibility of consumer choice, so a higher seller-buyer ratio increases the average quality of the goods that are chosen by buyers. This is because more sellers per buyer means that buyers can choose from a greater number of sellers (on average). With probability $1 - \alpha$, there are random, bilateral meetings and no consumer choice, so the seller-buyer ratio has no effect on the average quality of chosen goods. As a result, both the average quality of a chosen good and the average surplus depend directly on the seller-buyer ratio and also on the degree of consumer choice α. The surplus share of firms – our measure of market power – is also endogenous and depends on both the seller-buyer ratio and the degree of consumer choice α.

After buyers choose a seller with whom to trade, they choose the quantity of the good to purchase and make the corresponding payment. We focus on incentive-compatible direct revelation mechanisms that induce buyers to reveal their private information to their chosen seller. We establish the existence and uniqueness of equilibrium for any degree of consumer choice $\alpha \in (0, 1]$. There is only one active submarket in equilibrium, in which all sellers offer the same non-linear price schedule. For every realization of the buyer's utility shock, the price schedule specifies both the quantity traded and the corresponding payment in real dollars. Within any meeting, trades may or may not be liquidity constrained. Buyers may spend all of their money, some of their money, or none of their money.

After characterizing equilibrium, we examine the effects of consumer choice on market power. We find that market power can vary with the degree of consumer choice α in a non-monotonic manner. The source of this non-monotonicity is the fact that market power is driven by the generalized Hosios condition because prices are determined through competitive search. This condition says that firms' surplus share equals the matching elasticity plus the surplus elasticity (Mangin and Julien, 2021).

The two components of firms' surplus share, i.e. the matching elasticity and the surplus elasticity, reflect both the standard search externality that appears in random search models (Hosios, 1990), plus a novel *choice externality*. The choice externality arises when the degree of consumer choice $\alpha > 0$ and it reflects the fact that a higher seller-buyer ratio increases the average match surplus because buyers have more sellers to choose from, which shifts the endogenous distribution of chosen goods and increases average quality. Intuitively, firms' market power results from the fact that competitive search ensures that firms are compensated for their effect on both match creation (i.e. the number of trades) and on surplus creation (i.e. the average size of the trade surplus). With random matching and bilateral meetings, i.e. in the limit as $\alpha \to 0$, the distribution of chosen goods does not depend on the seller-buyer ratio, so the standard Hosios (1990) condition applies and the latter effect does not exist.

The fact that firms' market power is driven by both the search externality (through the matching elasticity) and the choice externality (through the surplus elasticity) means that market power varies with the degree of choice in a complex way. While the matching elasticity is always decreasing in the degree of choice (because it is decreasing in the seller-buyer ratio), the surplus elasticity can be either increasing or it can vary non-monotonically with the degree of choice α. In both cases, the overall effect on firms' market power is non-monotonic. At lower levels of consumer choice, an increase in choice α leads to an *increase* in firms' market power, while at higher levels of consumer choice, an increase in choice α leads to a *decrease* in firms' market power.

Next, we illustrate that consumer choice matters by quantifying the effect of choice on the welfare cost of inflation. To do so, we first calibrate the model to match data from Lucas and Nicolini (2015) on money demand in the U.S. from 1915-2008. For our baseline calibration, we target a retail markup of 30% as in Berentsen, Menzio, and Wright (2011), which implies that 54% of all meetings are ones in which consumers have a choice of seller. We estimate that the welfare cost of going from 0% to 10% inflation is equivalent to 0.93% of consumption at our baseline calibration.

To determine the effect of consumer choice on the welfare cost of inflation, we vary the degree of choice α and recalibrate the model using the same calibration strategy for the other parameters. In particular, we compare results for the full choice calibration (i.e. $\alpha = 1$) and the random matching calibration where all meetings are bilateral (i.e. the limit as $\alpha \to 0$). We estimate that the cost of increasing inflation from 0% to 10% is more than twice as high with full choice: 1.45% of consumption compared to 0.61% with random matching. Moreover, we find that the cost of inflation is strictly increasing in the degree of consumer choice α.

In our model, consumer choice makes inflation more costly because it *amplifies* the negative effects of inflation. With random matching, inflation is costly because buyers hold less money when inflation is higher, which leads to lower quantities traded and lower entry of sellers, which reduces the number of trades. When there is consumer choice, all of these effects continue to hold. However, there is an additional effect of inflation: lower seller entry directly reduces the average quality of chosen goods, which affects welfare by reducing the average match surplus directly (as well as indirectly through quantities). Similarly to the effect of choice on market power, this is because the distribution of chosen goods is endogenous and depends on the seller-buyer ratio.

Outline. Section 2 discusses the related literature. Section 3 describes the model. Section 4 describes the first-best allocation and discusses the effect of consumer choice on the first-best levels of both welfare and market power. Section 5 describes competitive search equilibrium, establishes the existence and uniqueness of equilibrium, and examines the effects of consumer choice on firms' equilibrium market power. Section 6 presents our baseline calibration, which is used for our estimates of the welfare cost of inflation in Section 7. Section 8 describes the results of some robustness exercises and Section 9 concludes. All proofs can be found in the Online Appendix.

2 Related literature

A theoretical contribution of this paper is to generalize and extend the basic framework in Mangin (2025) in two different ways. First, by introducing monetary exchange. Second, by incorporating a parameter that represents the *degree of consumer choice*. This generalization allows us to vary the extent to which consumers have a choice of seller prior to trading or are instead randomly matched. This generalization is important because it enables us to answer the two key questions in this paper: How does firms' market power vary with changes in the degree of consumer choice? How does the welfare cost of inflation vary with changes in the degree of consumer choice?

By contrast to Mangin (2025), buyers in our model may be either randomly matched or may potentially have a choice of seller, allowing us to nest both random matching *and* consumer choice. This approach has some similarities to Burdett and Judd (1983) search-theoretic models of imperfect competition in which buyers can either meet one seller or two (or more) sellers. In such models, the parameter governing the probability of a monopoly ("captive") versus competitive ("non-captive") meeting varies between zero and one; for example, see Menzio (2024). One key difference is that, in our model, sellers cannot influence how likely it is that they are chosen *within* a meeting; they can only influence how likely it is that a buyer chooses to search in their *submarket*. In our model, buyers' choice of submarket is driven by firms' decisions about prices, while buyers' choice of seller within a meeting is driven by utility shocks (not prices) because all sellers within a submarket set the same price-quantity schedule.

The distinction between consumer choice and random matching by buyers is also reminiscent of the distinction between informed and uninformed buyers in Lester (2011), which finds that having more informed buyers may either decrease or increase prices. Our results are similarly counter-intuitive but complementary. We find that firms' market power and markups can vary non-monotonically with the degree of consumer choice. In our model, *all* buyers observe price schedules and engage in directed or competitive search when choosing submarkets, but *within meetings* buyers either have a choice of seller *or* there is random matching without consumer choice.

This paper is also related to the literature on directed and competitive search. For a survey, see Wright et al. (2021). In particular, we contribute to the literature on directed or competitive search and private information, including Faig and Jerez (2005), Menzio (2007), Guerrieri (2008), Guerrieri, Shimer, and Wright (2010), Moen and Rosen (2011), and Davoodalhosseini (2019). This paper is also related to a literature that features many-on-one or multilateral meetings in monetary environments include Julien, Kennes, and King (2008) and Galenianos and Kircher (2008), and it is closely related to a literature that considers monetary environments featuring private information including Ennis (2008), and Faig and Jerez (2006), which builds on Faig and Jerez (2005), Dong and Jiang (2014), and Choi and Rocheteau (2023).

This paper is related to the large literature on the welfare cost of inflation. Rocheteau and Nosal (2017) provides a summary of estimates of the welfare cost of 10% inflation, which vary from 0.2% to 7.2% of consumption. Cooley and Hansen (1989) estimates the cost of 10% inflation is less than 0.5% of consumption, while Lucas (2000) estimates that it is less than 1%. Lagos and Wright (2005) finds that the cost of 10% inflation is between 3% and 5% of consumption in a monetary model with search and bargaining. In competitive search equilibrium, the cost of inflation is typically much lower than under bargaining, e.g. Rocheteau and Wright (2009) estimates the cost of 10% inflation is between 0.67% and 1.1% of consumption.[1] More recently, Bethune, Choi, and Wright (2020) obtains a relatively low estimate for the cost of inflation – around 1% or less – by identifying a positive market-composition effect of inflation.

In a related paper featuring monetary exchange in a search-theoretic model without private information, Dong (2010) considers the effect of product variety on the welfare cost of inflation when firms can invest to expand product variety. In contrast to our finding that the effect of consumer choice on the welfare cost of inflation is significant, Dong (2010) finds that the effect of endogenous product variety on the welfare cost of inflation is negligible in competitive search equilibrium.[2]

3 Model

Each time period $t \in \{0, 1, 2, ...\}$ is divided into two subperiods, day and night, as in Lagos and Wright (2005). During the day, there is a frictionless, centralized market and at night there is a frictional, decentralized market.

There is a continuum of agents who are either ex-ante identical *buyers* or ex-ante identical *sellers*. During the day all agents both produce and consume, but at night only

[1] Rocheteau and Wright (2009) use a slightly different formulation to calibrate the model in Rocheteau and Wright (2005). Instead of seller entry, agents decide whether to be buyers or sellers.

[2] In a related paper, Silva (2017) incorporates endogenous product variety into a monetary search model featuring monopolistic competition.

buyers wish to consume but cannot produce, and sellers do not wish to consume but can produce. The measure of buyers is fixed and equal to one. All buyers participate in the night market at zero cost, but sellers must decide whether to enter and pay a cost $\kappa > 0$. A subset of sellers of measure $n_t \in \mathbb{R}_+$ enter the night market. Since the measure of buyers is one, n_t is also the seller-buyer ratio.

The aggregate money supply at date t is $M_t \in \mathbb{R}_+$, which grows at a constant rate $\gamma \in \mathbb{R}_+$, i.e. $M_{t+1} = \gamma M_t$. Money is either injected into the economy ($\gamma > 1$) or withdrawn ($\gamma < 1$) by lump sum transfers during the day. We assume these transfers are to buyers only, and we restrict attention to policies where $\gamma \geq \beta$, where β is the discount factor. At the Friedman rule, where $\gamma = \beta$, we consider equilibria obtained by taking the limit as $\gamma \to \beta$ from above.

The price of goods in the day market is normalized to one and the relative price of money is denoted by ϕ_t. The aggregate real money supply is $Z_t \equiv \phi_t M_t$, and the real value of a quantity $\hat{m}_t$ of money held by an agent is $z_t \equiv \phi_t \hat{m}_t$.

We focus on steady-state equilibria where all of the aggregate real variables are constant. In steady state, we have $\phi_{t+1}/\phi_t = 1/\gamma$ because $M_{t+1}/M_t = \gamma$.

Prices in the night market are determined in competitive search equilibrium whereby "market makers" post terms of trade and agents choose which "submarket" to participate in. We discuss competitive search equilibrium in detail in Section 5.

Degree of consumer choice. In the decentralized market, there are two different possibilities with respect to how buyers can meet sellers.

With probability $\alpha \in (0, 1]$, the buyer has the possibility of *consumer choice* through a one-to-many meeting in which a buyer can meet either no sellers, one seller, or more than one seller. If the buyer meets at least one seller, the buyer can *choose* a single seller with whom they wish to trade. We refer to α as the *degree of consumer choice.*

With probability $1 - \alpha$, the buyer does not have consumer choice and can only meet a seller through random, bilateral meetings. Either they meet a single seller in a bilateral meeting, or they meet no seller in that period. We refer to the limiting case where $\alpha \to 0$ as *random matching* because buyers have no choice regarding sellers.

In this way, our environment can nest the standard approach in search-theoretic models of monetary exchange as the special limiting case where $\alpha \to 0$.

One-to-many meetings. With probability α, a buyer has the possibility of consumer choice because meetings are one-to-many. We assume that the probability a buyer meets $j \in \{0, 1, 2, ...\}$ sellers is given by a Poisson distribution with parameter n. That is, the expected number of sellers a buyer meets in a one-to-many meeting is equal to the seller-buyer ratio n, and the probability a buyer meets j sellers is

$$P_j(n) = \frac{n^j e^{-n}}{j!} \tag{1}$$

for all $j \in \{0, 1, 2, ...\}$. The endogenous probability $m(n)$ that a buyer has the opportunity to trade equals the probability that the buyer meets at least one seller, i.e. $m(n) = 1 - e^{-n}$. The probability a seller has the opportunity to trade is $m(n)/n$, which is just the probability that the seller is successfully chosen by a buyer.

Bilateral meetings. With probability $1 - \alpha$, a buyer has no possibility of consumer choice. Meetings are bilateral and matching is random. Consistent with the above, the endogenous probability $m(n)$ that a buyer has the opportunity to trade is $m(n) = 1 - e^{-n}$, but in this case the buyer can only meet exactly *one* seller. The probability that a seller has the opportunity to trade with a buyer is again equal to $m(n)/n$.

Distribution of utility shocks. The random utility shocks a are drawn from a bounded distribution with cdf G that is known to all agents. The realizations of the utility shocks are private information for the buyer. For convenience, we sometimes refer to the buyer's utility shock associated with a purchased good as the good's *quality*, although this is an idiosyncratic perception of quality that is specific to the buyer.

We assume that G is not degenerate and Assumption 1 is maintained throughout.

Assumption 1. *The distribution of utility shocks has a twice-differentiable cdf G, where $G' = g > 0$ and $G'' \leq 0$, and bounded support $A = [a_0, \bar{a}] \subseteq \mathbb{R}_+$.*

We also assume that the virtual valuation function of G is weakly increasing, a condition known as *regularity* in the mechanism design literature.[3] This assumption will later be used to prove the existence of equilibrium.

Assumption 2. *The distribution is regular, i.e. $\psi_G'(a) \geq 0$ where*

$$\psi_G(a) \equiv a - \frac{1 - G(a)}{g(a)}. \tag{2}$$

[3] This condition is weaker than both the increasing hazard rate condition and log-concavity.

Given the degree of consumer choice $\alpha \in (0, 1]$ and the seller-buyer ratio $n \in \mathbb{R}_+$, the distribution of the utility shocks for the goods actually *chosen* by buyers has cdf denoted by $\tilde{G}_\alpha : A \to [0, 1]$. This distribution is endogenous and depends on both the equilibrium seller-buyer ratio n and the actual choices made by buyers. For brevity, we refer to G simply as the *distribution of available goods* and $\tilde{G}_\alpha$ as the *distribution of chosen goods*. We discuss the distribution $\tilde{G}_\alpha$ further in Section 4.1.

Buyer and seller utility. On demand, sellers can produce any quantity $q \in \mathbb{R}_+$ of a divisible good and the cost of production is $c(q)$, where $c : \mathbb{R}_+ \to \mathbb{R}_+$. We assume that $c(0) = 0$, $c'(q) > 0$, and $c''(q) \geq 0$ for all $q > 0$. A buyer who consumes quantity q of a good with utility shock a receives utility $au(q)$, where $u : \mathbb{R}_+ \to \mathbb{R}_+$. We assume that $u(0) = 0$, $u'(0) = \infty$, $u'(q) > 0$, and $u''(q) < 0$ for all $q > 0$.

The instantaneous utility of a buyer who meets a seller at night is

$$U^b = \nu(x) - y + \beta E_{\tilde{G}_\alpha}(au(q_a)), \tag{3}$$

and the instantaneous utility of a seller who is chosen by a buyer at night is

$$U^s = \nu(x) - y - \beta E_{\tilde{G}_\alpha}(c(q_a)), \tag{4}$$

where x is the quantity consumed and y is the quantity produced during the day, q_a is the quantity consumed at night, a is the utility shock of the good consumed, and $\tilde{G}_\alpha$ is the *distribution of chosen goods*. We assume $\nu'(x) > 0$ and $\nu''(x) < 0$ for all x, and that there exists x^* such that $\nu'(x^*) = 1$.[4]

4 First-best allocation

Before studying competitive search equilibrium, we describe the first-best allocation. To do this, consider a planner that is constrained by the same search frictions and meeting technology as the decentralized market. That is, the planner faces probability $\alpha \in (0, 1]$ of a one-to-many meeting between a buyer and (possibly) many sellers and probability $1-\alpha$ of random matching with bilateral meetings. In terms of informational frictions, the planner has complete information about buyers' utility shocks.

[4]For now, we normalize $\nu(x^*) - x^* = 0$ as in Rocheteau and Wright (2005). Later, when we calibrate the model in Section 6, we will reverse this normalization.

Given the cost of seller entry $\kappa > 0$, the planner chooses a seller-buyer ratio n^*, a function $q^* : A \to \mathbb{R}_+$, and a distribution of chosen goods $\tilde{G}_\alpha : A \to [0,1]$ to maximize the total surplus created minus the total cost of seller entry, subject to the above constraints regarding search frictions. The planner solves the following problem:

$$\max_{n\in\mathbb{R}_+,\{q_a\}_{a\in A}} \left\{ m(n) \int_{a_0}^{\bar{a}} [au(q_a) - c(q_a)]\, d\tilde{G}_\alpha(a;n) - n\kappa \right\} \tag{5}$$

where $\tilde{G}_\alpha$ represents the planner's optimal choice of seller for each buyer.

Define $s_a \equiv au(q_a) - c(q_a)$, the trade surplus (or match surplus) for a good with utility shock (or "quality") a. Let q_a^* denote the first-best quantity of good a and define $s_a^* \equiv au(q_a^*) - c(q_a^*)$. Assume there is a (weakly) positive trade surplus at the lowest utility a_0. For any $\alpha \in (0,1]$, define the *expected trade surplus* by

$$\tilde{s}_\alpha(n; \{q_a\}_{a\in A}) \equiv \int_{a_0}^{\bar{a}} [au(q_a) - c(q_a)] d\tilde{G}_\alpha(a;n). \tag{6}$$

Throughout the paper, we sometimes suppress the dependence on $\{q_a\}_{a\in A}$ and let $\tilde{s}_\alpha(n)$ denote $\tilde{s}_\alpha(n; \{q_a\}_{a\in A})$ and $\tilde{s}'_\alpha(n)$ denote $\partial\tilde{s}(n)/\partial n$.

4.1 Distribution of chosen goods

Before presenting the first-best allocation, we derive the distribution of chosen goods $\tilde{G}_\alpha$ that results from the planner's optimal choices of seller for each buyer. This will turn out to be the distribution of chosen goods in equilibrium as well as for the first-best, so we refer to it simply as the *distribution of chosen goods*.

With probability α, the meeting is one-to-many and the planner chooses the seller which maximizes the trade surplus. The envelope theorem implies that s_a^* is increasing in a, so the planner chooses the seller with the highest utility shock for that buyer.[5]

Recalling that $P_j(n)$ is the probability a buyer meets j sellers when the seller-buyer ratio is n, the cdf of the maximum utility for the buyer – conditional on meeting at least one seller – is given by the following:

$$\frac{\sum_{j=1}^{\infty} P_j(n)(G(a))^j}{m(n)}. \tag{7}$$

[5]The envelope theorem implies $\frac{ds_a^*}{da} = \frac{\partial s_a}{\partial a}$, and $\frac{\partial s_a}{\partial a} = u(q_a^*) > 0$, so s_a^* is strictly increasing in a.

Given the meeting technology P_j is Poisson, this can be shown to be

$$\frac{e^{-n(1-G(a))} - e^{-n}}{1 - e^{-n}}. \tag{8}$$

With probability $1 - \alpha$, the meeting is bilateral and matching is random. The cdf of the utility of the chosen good in this case is just the cdf of available goods G.

Lemma 1. *For any degree of consumer choice $\alpha \in (0, 1]$, the distribution of chosen goods is given by*

$$\tilde{G}_\alpha(a; n) = \alpha \left(\frac{e^{-n(1-G(a))} - e^{-n}}{1 - e^{-n}} \right) + (1 - \alpha)G(a). \tag{9}$$

Therefore, $\tilde{a}_\alpha(n) = \alpha \tilde{a}_1(n) + (1 - \alpha)E_G(a)$ and $\tilde{s}_\alpha(n) = \alpha \tilde{s}_1(n) + (1 - \alpha)E_G(s_a)$.

Lemma 2 states some properties of the distribution of chosen goods and how it varies with changes in the seller-buyer ratio n. In particular, Lemma 2 says that the distribution of chosen goods first-order stochastically dominates the distribution of available goods, and the average quality of a *chosen* good, $\tilde{a}_\alpha(n) \equiv \int_{a_0}^{\bar{a}} a d\tilde{G}_\alpha(a; n)$, is greater than the average quality of an *available* good, $E_G(a)$.

Lemma 2. *For any degree of consumer choice $\alpha \in (0, 1]$,*

1. *In the limit as $n \to 0$, we have $\tilde{G}_\alpha(a; n) \to G(a)$ and $\tilde{a}_\alpha(n) \to E_G(a)$.*

2. *In the limit as $n \to \infty$, we have $\tilde{G}_\alpha(a; n) \to (1 - \alpha)G(a)$ for all $a \in [a_0, \bar{a})$ and $\tilde{a}_\alpha(n) \to \alpha \bar{a} + (1 - \alpha)E_G(a)$.*

3. *The distribution of chosen goods $\tilde{G}_\alpha(a; n)$ first-order stochastically dominates the distribution of available goods $G(a)$ and $\tilde{a}_\alpha(n) > E_G(a)$.*

4. *If $n_1 > n_2$, the distribution $\tilde{G}_\alpha(a; n_1)$ first-order stochastically dominates the distribution $\tilde{G}_\alpha(a; n_2)$ and $\tilde{a}_\alpha(n_1) > \tilde{a}_\alpha(n_2)$.*

5. *For any $f : A \to \mathbb{R}_+$ where $f' > 0$, $\tilde{f}'_\alpha(n) > 0$ where $\tilde{f}_\alpha(n) \equiv \int_{a_0}^{\bar{a}} f(a) d\tilde{G}_\alpha(a; n)$.*

As a special case, Part 5 of Lemma 2 implies that $\tilde{a}'_\alpha(n) > 0$, i.e. the average quality of a chosen good is increasing in the seller-buyer ratio n. Intuitively, average quality is increasing in the seller-buyer ratio n because more sellers per buyer means greater choice of seller and a higher expected quality of the chosen good. We know that s_a is increasing in a, so Part 5 of Lemma 2 also implies that $\tilde{s}'_\alpha(n) > 0$.[6]

Lemma 3 presents some additional properties of the distribution of chosen goods and shows how this distribution varies with changes in the degree of choice α. Recall that we can interpret the limit as $\alpha \to 0$ as a standard monetary search environment with random, bilateral meetings in which buyers can either trade or wait but there is no consumer choice. In the case where $\alpha \to 0$, neither the distribution of chosen goods nor the average quality of a chosen good depends on the seller-buyer ratio n.

Lemma 3. *For any seller-buyer ratio $n > 0$,*

1. *In the limit as $\alpha \to 0$, we have $\tilde{G}_\alpha(a; n) \to G(a)$ and $\tilde{a}_\alpha(n) \to E_G(a)$.*
2. *In the limit as $\alpha \to 1$ (i.e.* full choice*),*

$$\tilde{G}_\alpha(a; n) = \frac{e^{-n(1-G(a))} - e^{-n}}{1 - e^{-n}}. \tag{10}$$

3. *If $\alpha_1 > \alpha_2$, the distribution $\tilde{G}_{\alpha_1}(a; n)$ first-order stochastically dominates the distribution $\tilde{G}_{\alpha_2}(a; n)$ and $\tilde{a}_{\alpha_1}(n) > \tilde{a}_{\alpha_2}(n)$.*
4. *For any $f : A \to \mathbb{R}_+$ where $f' > 0$ and $\tilde{f}_\alpha(n) \equiv \int_{a_0}^{\bar{a}} f(a) d\tilde{G}_\alpha(a; n)$, we have*

$$\frac{\partial \tilde{f}_\alpha(n)}{\partial \alpha} > 0. \tag{11}$$

For example, Part 4 of Lemma 3 implies that $\frac{\partial \tilde{a}_\alpha(n)}{\partial \alpha} > 0$, i.e. when the degree of consumer choice α increases, the average quality $\tilde{a}_\alpha(n)$ of a chosen good increases. Because we know that s_a^* is increasing in a, it also implies that $\frac{\partial \tilde{s}_\alpha(n)}{\partial \alpha} > 0$, i.e. when α increases, the expected trade surplus $\tilde{s}_\alpha(n)$ increases. Intuitively, this is because a higher α means that consumers are more likely to be able to choose the good that will deliver the highest quality, rather than simply meeting one seller at random, thereby increasing both the average quality and the expected trade surplus.

4.2 First-best allocation

We are now in a position to describe the first-best allocation. Before we do this, we need Assumption 3 to ensure existence of the first-best seller-buyer ratio $n^* > 0$.

[6]We have seen this holds for the planner's solution s_a^*, but it will also hold for the equilibrium s_a.

Assumption 3. *The cost of entry is not too high:* $E_G[au(q_a^*) - c(q_a^*)] > \kappa$.

Proposition 1 states that there exists a unique first-best allocation $(n^*, \{q_a^*\}_{a \in A})$ with $n^* > 0$ and provides the necessary conditions.[7]

Proposition 1. *There exists a unique first-best allocation* $(n^*, \{q_a^*\}_{a \in A})$ *and it satisfies the following:*

1. *For any* $a \in A$*, the quantity* $q_a^* > 0$ *solves*

$$au'(q_a^*) = c'(q_a^*). \tag{12}$$

2. *For any* $\alpha \in (0, 1]$*, the seller-buyer ratio* $n^* > 0$ *satisfies*

$$m'(n^*)\tilde{s}_\alpha(n^*; \{q_a^*\}_{a \in A}) + m(n^*)\tilde{s}'_\alpha(n^*; \{q_a^*\}_{a \in A}) = \kappa. \tag{13}$$

3. *For any* $\alpha \in (0, 1]$*, the distribution of chosen goods is given by (9).*

4. *The seller-buyer ratio* n^* *is strictly increasing in the degree of choice* α*.*

Observe that while the first-best quantity q_a^* is standard and does not depend on either the seller-buyer ratio n^* or the degree of consumer choice α, the endogenous distribution of chosen goods $\tilde{G}_\alpha$ *does* depend on the degree of consumer choice α and therefore so does the first-best seller-buyer ratio n^*. In particular, the first-best seller-buyer n^* is increasing in the degree of consumer choice α.

4.3 Effect of choice on welfare

For any given cost of entry $\kappa > 0$ that satisfies Assumption 3, and any given degree of consumer choice $\alpha \in (0, 1]$, welfare at the first-best allocation is defined by

$$W_\alpha^* \equiv m(n^*)\tilde{s}_\alpha(n^*; \{q_a^*\}_{a \in A}) - n^*\kappa. \tag{14}$$

Since we have already shown that the match surplus s_a^* is increasing in quality a, Part 4 of Lemma 3 implies that $\frac{\partial \tilde{s}_\alpha(n)}{\partial \alpha} > 0$ (as discussed above). Therefore, we have $\frac{\partial W_\alpha^*}{\partial \alpha} > 0$ because the expected trade surplus $\tilde{s}_\alpha(n^*; \{q_a^*\}_{a \in A})$ is the only component of welfare that depends directly on the degree of choice α. Applying the envelope theorem, we obtain the following result regarding the effect of consumer choice on welfare.

[7]Note that it follows from our previous assumptions that, for all $a \in A$, there exists a unique $q_a^* \in \mathbb{R}_+$ such that $au'(q_a^*) = c'(q_a^*)$.

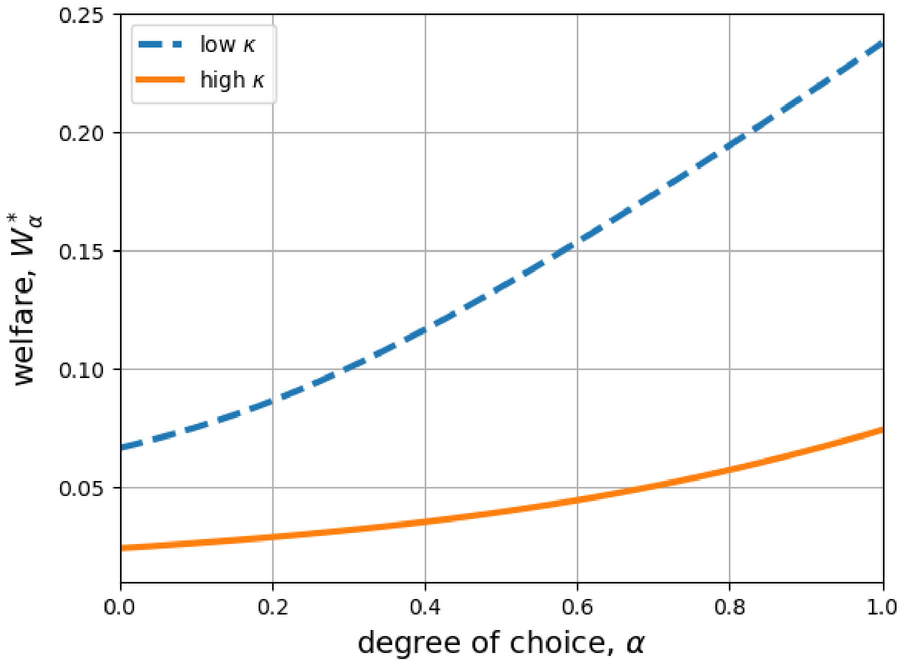

Figure 1: Welfare as a function of degree of consumer choice α for different κ

Proposition 2. *At the first-best allocation, welfare W_α^* is strictly increasing in the degree of consumer choice α and welfare W_α^* is maximized at $\alpha = 1$ (i.e. full choice).*

Unsurprisingly, welfare is strictly increasing in the degree of consumer choice α. It is intuitive that an increase in the degree of choice by consumers leads to an increase in the expected trade surplus and therefore an increase in welfare.

Figure 1 illustrates Proposition 2 for both low and high entry cost κ. This figure shows that welfare increases with the degree of consumer choice α, and it does so at a steeper rate when the entry cost κ is lower. This is because a lower entry cost κ leads to a higher first-best seller-buyer ratio n^*. A higher seller-buyer ratio n^* amplifies the positive effect of greater consumer choice α on welfare because it increases the difference between the distribution of available goods G and the distribution of chosen goods $\tilde{G}_\alpha$ because buyers in one-to-many meetings can choose from more sellers.

4.4 Effect of choice on first-best market power

We define firms' *market power* as firms' surplus share. Intuitively, when firms have more market power, they are able to extract a higher share of the total surplus relative to consumers. While it is more standard to define firms' market power as the average markup, in a competitive search model the planner's solution gives positive surplus share to firms and this is *efficient*. By defining firms' market power as firms' surplus share, we are therefore able to consider *both* the first-best market power, which is the efficient degree of market power, *and* the equilibrium market power, which is firms' market power in equilibrium. We will later discuss in Section 5.2 the relationship between this measure of market power and the average markup in equilibrium, and show that our main findings generalize to this standard measure of market power.

Generalized Hosios condition. The first-order condition for n^*, which is given by (13), can be rearranged into the form of the generalized Hosios condition derived in Mangin and Julien (2021). Writing it in this form allows us to decompose firms' market power (or surplus share) into two key components, the first of which is standard and the second of which is novel and arises due to the presence of consumer choice.

Defining the *matching elasticity* by $\eta_m(n) \equiv m'(n)n/m(n)$ and the *surplus elasticity* by $\eta_{s_\alpha}(n) \equiv \tilde{s}'_\alpha(n)n/\tilde{s}_\alpha(n)$, we obtain the generalized Hosios condition:

$$\underbrace{\eta_m(n^*)}_{\text{matching elasticity}} + \underbrace{\eta_{s_\alpha}(n^*)}_{\text{surplus elasticity}} = \underbrace{\frac{n^*\kappa}{m(n^*)\tilde{s}_\alpha(n^*)}}_{\text{firms' surplus share}}. \tag{15}$$

In the limit as $\alpha \to 0$, there is no consumer choice and all meetings are bilateral, as in standard monetary search models. In this case, the average trade surplus does not depend directly on the seller-buyer ratio n, so the surplus elasticity $\eta_{s_\alpha}(n)$ is zero. In the limit as $\alpha \to 0$, we therefore obtain the standard Hosios (1990) condition, which states that the level of seller entry n is constrained efficient only if firms' surplus share equals the matching elasticity $\eta_m(n)$ because this reflects the appropriate level of compensation for firms' contribution to the total surplus by creating matches.

With consumer choice, i.e. for any degree of choice $\alpha > 0$, the surplus elasticity is non-zero. We have seen that $\tilde{s}'_\alpha(n) > 0$ and therefore the surplus elasticity $\eta_{s_\alpha}(n)$ is strictly positive. Intuitively, more sellers per buyer means greater choice for buyers, thus increasing the average surplus. This means there is a positive externality arising from the effect of seller entry on the average surplus when there is consumer choice. This positive externality – which we call the *choice externality* – is additional to the standard search externality arising from the effect of seller entry on the matching probability $m(n)$ for buyers. This choice externality is reflected in the surplus elasticity $\eta_{s_\alpha}(n)$. When the generalized Hosios condition (15) holds in equilibrium, we have constrained efficiency of seller entry because both the standard search externality and the choice externality are internalized and firms are compensated for their contribution to both creating matches and creating additional surplus per match.

Let $\rho_\alpha(n^*)$ denote firms' surplus share at the first-best allocation. For any given cost of entry $\kappa > 0$ that satisfies Assumption 3, and any given degree of consumer choice $\alpha \in (0, 1]$, firms' surplus share at the first-best is given by

$$\rho_\alpha(n^*) = \frac{n^*\kappa}{m(n^*)\tilde{s}_\alpha(n^*)}. \tag{16}$$

Consider the fact that in equilibrium there will be a zero profit condition for firms and therefore the numerator on the right-hand side of (16) will be equal to firms' expected profits, while the denominator on the right of (16) is equal to the total surplus. Therefore, we can interpret this term as representing firms' surplus share.

Applying condition (15), firms' *market power* at the first-best is given by

$$\underbrace{\rho_\alpha(n^*)}_{\text{firms' market power}} = \underbrace{\eta_m(n^*)}_{\text{matching elasticity}} + \underbrace{\eta_{s_\alpha}(n^*)}_{\text{surplus elasticity}}. \tag{17}$$

Before we discuss the effect on market power of the degree of consumer choice, it is useful to first discuss the effect of the seller-buyer ratio on market power.

Effect of seller-buyer ratio on market power. If all meetings are bilateral (i.e. in the special case of our model where $\alpha \to 0$), firms' market power is equal to the matching elasticity $\eta_m(n^*)$. In this case, market power $\rho_\alpha(n^*)$ varies between zero and one and it is decreasing in the seller-buyer ratio n. In the limit as the seller-buyer ratio goes to zero, firms' market power goes to one. In the limit as the seller-buyer ratio goes to infinity, firms' market power goes to zero.[8] This fits with our intuitive understanding of market power: more competition between firms decreases market power. This is because market power depends *only* on the seller-buyer ratio as $\alpha \to 0$.

Suppose instead there is full consumer choice, i.e. $\alpha = 1$. Firms' market power is equal to the matching elasticity $\eta_m(n^*)$ plus the surplus elasticity $\eta_{s_\alpha}(n^*)$. In this case, it is also true that $\rho_\alpha(n^*)$ varies between zero and one. Similarly, it is also true that, in the limit as the seller-buyer ratio goes to zero, firms' market power goes to one. In the limit as the seller-buyer ratio goes to infinity, firms' market power goes to zero. This

[8]It is well known that the matching elasticity $\eta_m(n)$ is decreasing in n, and $\eta_m(n) \to 1$ as $n \to 0$, and $\eta_m(n) \to 0$ as $n \to \infty$, given that we have $m(n) = 1 - e^{-n}$.

again fits with our intuitive understanding of market power.

We summarize these intuitive results regarding market power in Lemma 4.

Lemma 4. *For any degree of consumer choice* $\alpha \in (0, 1]$,

1. *Firms' market power* $\rho_\alpha(n^*)$ *at the first-best varies between zero and one.*
2. *In the limit as the seller-buyer ratio* $n \to 0$*, firms' market power goes to one.*
3. *In the limit as the seller-buyer ratio* $n \to \infty$*, firms' market power goes to zero.*

Without consumer choice, i.e. as $\alpha \to 0$, firms' market power $\rho_\alpha(n^*)$ is decreasing in the seller-buyer ratio n. However, with consumer choice, firms' market power $\rho_\alpha(n^*)$ depends on n in a more complex way. To see this, consider the following two effects:

$$\frac{d\rho_\alpha(n^*)}{dn} = \underbrace{\frac{d}{dn}\eta_m(n^*)}_{\text{negative effect } (<0)} + \underbrace{\frac{d}{dn}\eta_{s_\alpha}(n^*).}_{\text{positive or negative effect } (>0 \text{ or } <0)} \tag{18}$$

In the limit as $n \to 0$, the distribution of chosen goods $\tilde{G}_\alpha \to G$ by Lemma 2. Therefore, in this limit, the average match surplus no longer depends on the seller-buyer ratio and $\eta_{s_\alpha}(n) \to 0$. Similarly, in the limit as $n \to \infty$, the distribution of chosen goods $\tilde{G}_\alpha$ converges to a degenerate distribution at the highest quality $\overline{a}$. Therefore, the average surplus again no longer depends on the seller-buyer ratio and $\eta_{s_\alpha}(n) \to 0$.

While the matching elasticity $\eta_m(n)$ is decreasing in n, the surplus elasticity $\eta_{s_\alpha}(n)$ is *non-monotonic* in n. This is why the second term above can be either positive or negative. At first, this elasticity is increasing for low levels of n, but it is eventually decreasing for sufficiently high n. Intuitively, the reason why the surplus elasticity varies *non-monotonically* with the seller-buyer ratio this is the fact that the distribution of chosen goods is no longer fixed and exogenous, but it is instead endogenous with consumer choice and depends on the seller-buyer ratio. Market power is driven by the competitive effect of the seller-buyer ratio, but it also driven by the effect of the seller-buyer ratio on the endogenous distribution of chosen goods, which determines the surplus elasticity (i.e. the measure of how firm entry affects the average surplus).

Effect of choice on market power. The effects on firms' market power of changes in the choice parameter α are complex. Not only are there two components (the matching elasticity and the surplus elasticity) but there is both an *indirect* effect through

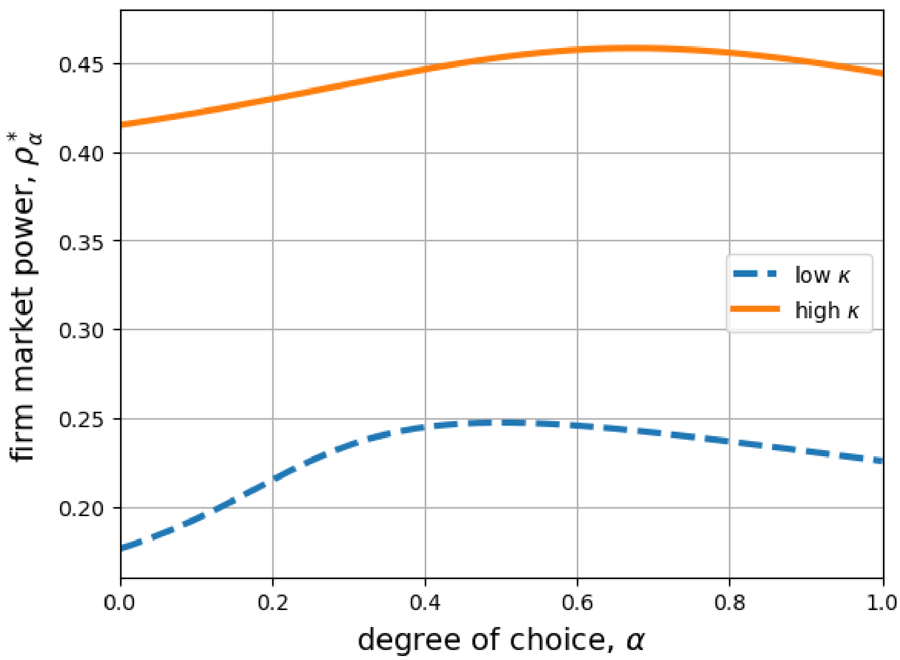

Figure 2: Firms' market power at first best as a function of choice α

n and a *direct* effect of the choice parameter α on the surplus elasticity $\eta_{s_\alpha}(n)$. To understand better what is driving the behavior of market power with respect to the degree of choice α, we can decompose the effect of choice into two components:

$$(19) \qquad \frac{d\rho_\alpha(n^*)}{d\alpha} = \underbrace{\frac{d}{dn}\eta_m(n^*)\frac{dn^*}{d\alpha}}_{\text{negative effect } (<0)} + \underbrace{\frac{\partial}{\partial n}\eta_{s_\alpha}(n^*)\frac{dn^*}{d\alpha} + \frac{\partial}{\partial \alpha}\eta_{s_\alpha}(n^*)}_{\text{positive or negative effect } (>0 \text{ or } <0)}.$$

We can think of the effect of consumer choice on the matching elasticity (i.e. sellers' contribution to the number of matches created) as capturing the "standard" effect reflected in the Hosios condition. The effect of choice α on the matching elasticity $\eta_m(n^*)$ is always negative because this elasticity is decreasing in n, and n^* is increasing in α by Proposition 1. However, the effect of choice α on the surplus elasticity (which reflects sellers' contribution to the average surplus per match) is more complex. The *direct* effect of choice α on the surplus elasticity is positive because $\frac{\partial}{\partial\alpha}\eta_{s_\alpha}(n^*) > 0$, but the overall effect of choice can be either positive or negative.[9] This is because, as we saw above, a higher degree of choice increases the seller-buyer ratio, but the effect of

[9]Applying Lemma 1, we can use the fact that $\tilde{s}_\alpha(n) = E_G(s_a) + \alpha[\tilde{s}_1(n) - E_G(s_a)]$ to derive the surplus elasticity $\eta_{s_\alpha}(n^*)$ and show that $\frac{\partial}{\partial\alpha}\eta_{s_\alpha}(n^*) > 0$.

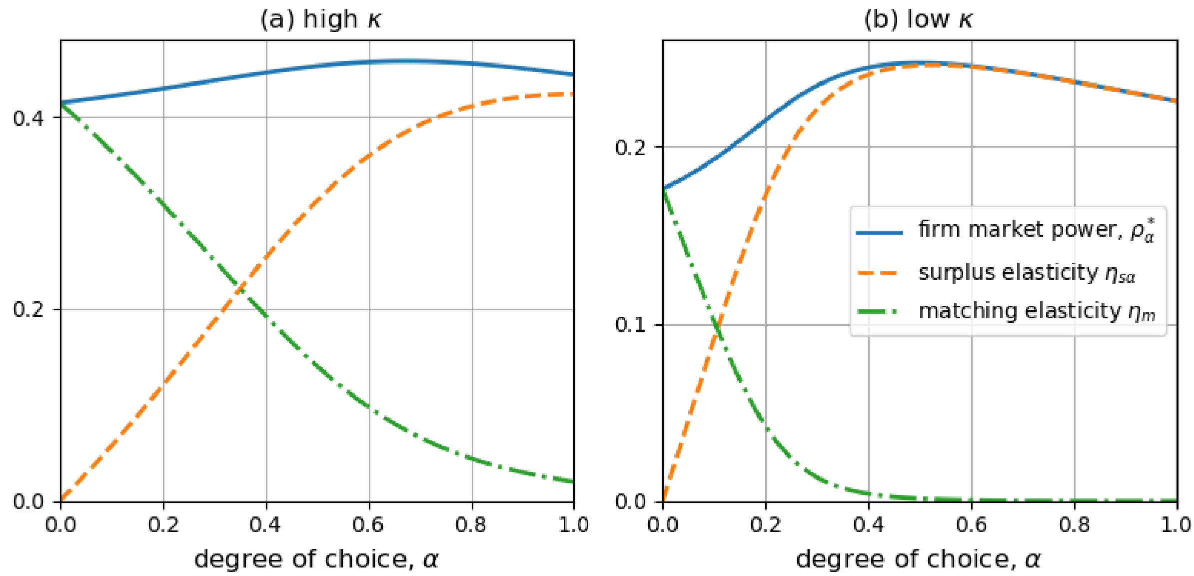

Figure 3: Decomposition of firms' market power at first best

the seller-buyer ratio n on the surplus elasticity can be either positive or negative.

Figure 2 shows that firms' market power can vary *non-monotonically* with the degree of consumer choice α. For this example, we consider a distribution of utility shocks G that is uniform on $[0, 1]$ and two different values of κ (high and low). For lower levels of consumer choice α, firms' market power is increasing in α, but for higher levels of α firms' market power is decreasing in α. This is surprising and counter-intuitive. One might expect that a greater degree of consumer choice would always lead to an increase in buyers' market power and a *decrease* in firms' market power.

Figure 3 decomposes the effect of choice on firms' market power into these two key components for the two examples in Figure 2. In both examples, the matching elasticity $\eta_m(n^*)$ is strictly decreasing in choice α. Also, for both examples, the surplus elasticity is increasing for lower levels of α. As $\alpha \to 0$, we have $\eta_{s_\alpha}(n^*) \to 0$, thus for low $\alpha > 0$ we must have $\eta_{s_\alpha}(n^*)$ increasing at first. As the degree of choice α increases further, the effect of choice on the average surplus is stronger, leading $\eta_{s_\alpha}(n^*)$ to increase further still. The fact that an increase in α also increases n^* reinforces this effect. Overall, the effect of the surplus elasticity dominates and firms' market power is increasing in the degree of consumer choice for lower levels of α in both examples.

In both examples, firms' market power is non-monotonic overall in the degree of choice α, but for different reasons. As α increases to higher levels, there are two alternatives depending on the level of n^*. In example (a) on the left of Figure 3, the surplus elasticity continues to increase for higher levels of α, albeit at a slower rate. As a result, the effect of the matching elasticity starts to dominate. Since n^* is relatively low (because κ is high), the matching elasticity is still decreasing in α relatively sharply through its effect on n^*. Therefore, firms' market power is decreasing for higher levels of α. Overall, firms' market power is *non-monotonic* in the degree of choice α.

In example (b) on the right of Figure 3, the seller-buyer ratio n^* is relatively high (because κ is low), so the matching elasticity $\eta_m(n^*)$ goes to zero quite rapidly as α increases. As a result, the effect of the surplus elasticity completely dominates for higher levels of α. As n^* becomes higher as α increases, the surplus elasticity eventually starts to fall as we move towards $\alpha = 1$. This is because, as $\alpha \to 1$, all chosen goods are the highest quality $\bar{a}$ in the limit as n becomes large, which weakens the effect of choice as $\alpha \to 1$ and n^* is sufficiently high. As a result, the surplus elasticity $\eta_{s_\alpha}(n^*)$ starts to decrease as α increases further, which pulls firms' market power downwards. Again, firms' market power varies *non-monotonically* with the degree of choice α.

5 Competitive search equilibrium

Following Rocheteau and Wright (2005), we model competitive search by assuming there are agents called "market makers" who can open submarkets by posting terms of trade. Specifically, market makers post contracts $\{(q_a, d_a)\}_{a \in A}$ which specify the quantity of the good q_a and the payment in real dollars d_a *contingent on the buyer's utility shock for their chosen seller.* Market makers take into account the relationship between the posted terms of trade and the expected seller-buyer ratio n.

Buyers and sellers choose which submarket to enter. Buyers and sellers who enter a submarket commit to trading at the terms specified within that submarket. Within each submarket, there are search frictions. Meetings take place, buyers choose sellers within meetings, and trade occurs as described in Section 3.

Within matches (i.e. between a buyer and their chosen seller), buyers' utility shocks are private information which is not observed by the seller. However, buyers may choose to reveal their private utility shocks to their chosen seller through their choice of contract (q_a, d_a). By the revelation principle, it is without loss of generality to focus on incentive-compatible direct mechanisms $\{(q_a, d_a)\}_{a \in A}$ that induce buyers to truthfully reveal their private information to their chosen sellers.

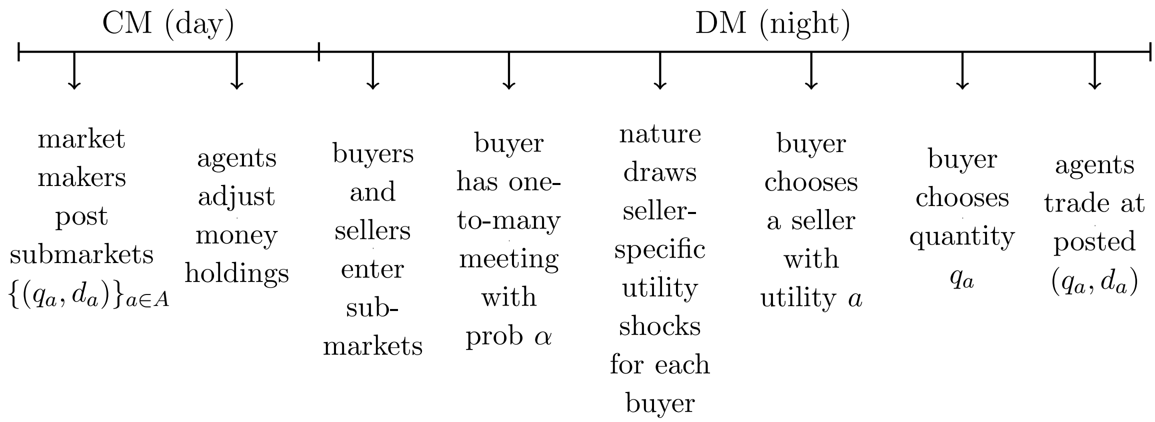

Figure 4: Timing of a representative period

At the beginning of each day, market makers post submarkets $\{(q_a, d_a)\}_{a \in A}$ that will open that night, implying an expected n for each submarket. During the day, agents adjust their real money holdings in the centralized market (CM), and then choose a submarket in which to trade at night, taking into account the expected seller-buyer ratio n in that submarket. At night, agents trade in the decentralized market (DM) through the frictional meeting process in their chosen submarket.

Centralized market. Let W^b and W^s denote the value functions for buyers and sellers, respectively, in the day market (CM). Let V^b and V^s denote the value functions for buyers and sellers, respectively, in the decentralized night market (DM).

In the CM, a buyer with real money holdings z solves:

$$W^b(z) = \max_{\hat{z}, x, y \in \mathbb{R}_+} \{\nu(x) - y + \beta V^b(\hat{z})\}, \tag{20}$$

subject to $\hat{z} + x = z + T + y$, where T is her real transfer and $\hat{z}$ is the real balance carried into that period's decentralized market. Substituting into (20) yields

$$W^b(z) = z + T + \max_{\hat{z}, x \in \mathbb{R}_+} \{\nu(x) - x - \hat{z} + \beta V^b(\hat{z})\}. \tag{21}$$

Thus, the buyer's $\hat{z}$ is independent of z, and $W^b(z) = z + W^b(0)$, which is linear.

Similarly, a seller with real balance z_s in the centralized market solves:

$$W^s(z_s) = \max_{\hat{z},x,y\in\mathbb{R}_+} \left\{ \nu(x) - y + \beta \max \left[V^s(\hat{z}), W^s\left(\frac{\hat{z}}{\gamma}\right)\right]\right\}, \tag{22}$$

subject to $\hat{z} + x = z_s + y$. Substituting into (22), we obtain

$$W^s(z_s) = z_s + \max_{\hat{z},x\in\mathbb{R}_+} \left\{ \nu(x) - x - \hat{z} + \beta \max \left[V^s(\hat{z}), W^s\left(\frac{\hat{z}}{\gamma}\right)\right]\right\}. \tag{23}$$

Thus, the seller's $\hat{z}$ is independent of z_s, and $W^s(z_s) = z_s + W^s(0)$.

Decentralized market. The value functions V^b and V^s depend on the distribution of chosen goods, which is endogenous and depends on the degree of consumer choice α, the seller-buyer ratio n, and buyers' choices. In any meeting, the buyer chooses the seller that maximizes $v_a \equiv au(q_a) - d_a/\gamma$, the buyer's ex post trade surplus. For any $\alpha \in (0, 1]$, the equilibrium distribution of chosen goods $\tilde{G}_\alpha$ is given by buyers' optimal choices of sellers. We later verify this equilibrium distribution is given by (9).

Let Ω denote the set of open submarkets, where each submarket $\omega \in \Omega$ is characterized by $(\{(q_a, d_a)\}_{a\in A}, n)$. For a seller in the decentralized night market,

$$V^s(z_s) = \max_{\omega\in\Omega} \left\{ \begin{array}{c} \frac{m(n)}{n} \int_{a_0}^{\bar{a}} \left[-c(q_a) + W^s\left(\frac{z_s+d_a}{\gamma}\right)\right] d\tilde{G}_\alpha(a;n) \\ + \left[1 - \frac{m(n)}{n}\right] W^s\left(\frac{z_s}{\gamma}\right) - \kappa \end{array} \right\}. \tag{24}$$

It is straightforward to verify that the seller's choice of real balances is $\hat{z} = 0$.

For a buyer in the decentralized night market,

$$V^b(z) = \max_{\omega\in\Omega} \left\{ \begin{array}{c} m(n) \int_{a_0}^{\bar{a}} \mathbf{1}_a \left[au(q_a) + W^b\left(\frac{z-d_a}{\gamma}\right)\right] d\tilde{G}_\alpha(a;n) \\ + \left[1 - m(n) \int_{a_0}^{\bar{a}} \mathbf{1}_a \, d\tilde{G}_\alpha(a;n)\right] W^b\left(\frac{z}{\gamma}\right) \end{array} \right\} \tag{25}$$

where $\mathbf{1}_a$ is an indicator function that is equal to one if $z \geq d_a$ and zero otherwise. Using $W^b(z) = z + W^b(0)$, we obtain

$$V^b(z) = \max_{\omega\in\Omega} \left\{ m(n) \int_{a_0}^{\bar{a}} \mathbf{1}_a \left[au(q_a) - \frac{d_a}{\gamma}\right] d\tilde{G}_\alpha(a;n) + \frac{z}{\gamma} + W^b(0)\right\}. \tag{26}$$

Thus, the buyer's choice of z from (21) is given by

$$\max_{z\in\mathbb{R}_+} \left\{ -z + \beta \max_{\omega\in\Omega} \left\{ m(n) \int_{a_0}^{\bar{a}} \left[au(q_a) - \frac{d_a}{\gamma}\right] d\tilde{G}_\alpha(a;n) + \frac{z}{\gamma}\right\}\right\} \tag{27}$$

subject to the liquidity constraint, $d_a \leq z$ for all $a \in A$.

Defining $i \equiv \frac{\gamma-\beta}{\beta}$, the nominal interest rate, the above problem is equivalent to

$$\max_{z\in\mathbb{R}_+,\ \omega\in\Omega} \left\{ m(n) \int_{a_0}^{\bar{a}} \left[au(q_a) - \frac{d_a}{\gamma} \right] d\tilde{G}_\alpha(a;n) - i\frac{z}{\gamma} \right\}, \tag{28}$$

subject to $d_a \leq z$ for all $a \in A$ plus the constraint that a submarket with posted contracts $\{(q_a, d_a)\}_{a\in A}$ will attract measure n of sellers per buyer, where n satisfies

$$\frac{m(n)}{n} \int_{a_0}^{\bar{a}} \left[-c(q_a) + \frac{d_a}{\gamma} \right] d\tilde{G}_\alpha(a;n) \leq \kappa \tag{29}$$

and $n \geq 0$ with complementary slackness.

As a result of buyers' private information, we need to impose two additional constraints on problem (28). The individual rationality (IR) constraint states that buyers must receive a (weakly) positive ex post trade surplus, or trade will not occur. The IR constraint for buyers is given by

$$au(q_a) - \frac{d_a}{\gamma} \geq 0 \tag{30}$$

for all $a \in A$. The incentive compatibility (IC) constraint states that a buyer with utility shock a cannot do better by choosing another contract $(q_{a'}, d_{a'})$ instead of (q_a, d_a). The IC constraint is given by

$$au(q_a) - \frac{d_a}{\gamma} \geq au(q_{a'}) - \frac{d_{a'}}{\gamma} \tag{31}$$

for all utility shocks $a, a' \in A$.

5.1 Existence, uniqueness, and characterization

We can now define competitive search equilibrium. We restrict attention to monetary equilibria with positive money holdings $z > 0$ and positive entry $n > 0$. We later prove that there is a unique solution to the market makers' problem and there is therefore only one active submarket in equilibrium. In anticipation of this result, we simply denote equilibrium by $(\{(q_a, d_a)\}_{a\in A}, z, n)$ and define it as follows.

Definition 1. *For any degree of consumer choice $\alpha \in (0,1]$, a* competitive search equilibrium *is a list $(\{(q_a, d_a)\}_{a\in A}, z, n)$ and a distribution of chosen goods $\{\tilde{G}_\alpha(a;n)\}_{a\in A}$ where $(q_a, d_a) \in \mathbb{R}_+^2$ for all $a \in A$, $\tilde{G}_\alpha(a;n) \in [0,1]$ for all $a \in A$, and $z, n \in \mathbb{R}_+ \backslash \{0\}$,* *such that* $(\{(q_a, d_a)\}_{a\in A}, z, n)$ *maximizes (28) subject to constraint (29), the liquidity constraint* $d_a \le z$ *for all* $a \in A$*, plus the IR constraint (30) and the IC constraint (31), and* $\{\tilde{G}_\alpha(a; n)\}_{a\in A}$ *represents buyers' optimal choices of sellers.*

Before presenting Proposition 3, it is helpful to define $\rho(a; n) \equiv 1 - \tilde{G}_\alpha(a; n)$, the probability that a chosen good has quality greater than a. We also define $\varepsilon_\rho(a; n) \equiv -a\rho'(a; n)/\rho(a; n)$, the elasticity of $\rho(a; n)$ with respect to a, where $\rho'(a; n) \equiv \frac{\partial \rho(a;n)}{\partial a}$.

We can now present our main result, which establishes the existence and uniqueness of equilibrium and provides a characterization. For the existence of equilibrium, we need to ensure that the entry cost κ is not "too high".

For simplicity, we also assume that $a_0 = 0$ for the remainder of the paper.

Assumption 4. *The cost of entry is not too high:* $E_G[au(q_a^0) - c(q_a^0)] > \kappa$.

In the Online Appendix, we make this condition precise by showing how to calculate $q_a^0 \equiv \lim_{n\to 0} q_a(n)$ in terms of the distribution of utility shocks G.

Proposition 3. *For any* $\alpha \in (0, 1]$ *and* $i > 0$*, if the cost of entry* κ *is not too high, there exists a unique competitive search equilibrium and it satisfies:*

1. *No-trade range. For any* $a \in [a_0, a_b]$, $q_a = 0$ *and* $d_a = 0$.
2. *Unconstrained range. For any* $a \in (a_b, a_c]$*, the quantity* $q_a > 0$ *solves:*

   $$(a - \phi(a; n))u'(q_a) = c'(q_a) \tag{32}$$

   where

   $$\phi(a; n) = \left(1 - \frac{1}{\delta}\right)\left(\frac{1 - \tilde{G}_\alpha(a; n)}{\tilde{g}_\alpha(a; n)}\right) - \left(\frac{1}{\delta}\right)\frac{i}{m(n)\tilde{g}_\alpha(a; n)} \tag{33}$$

   and

   $$\delta = \frac{1}{1 - \varepsilon_\rho(a_b; n)}\left(1 + \frac{i}{m(n)\rho(a_b; n)}\right). \tag{34}$$

   Also, we have

   $$\frac{d_a}{\gamma} = au(q_a) - \int_{a_0}^{a} u(q_x)dx. \tag{35}$$

3. *Liquidity constrained range. For any* $a \in [a_c, \bar{a}]$, $q_a = q_{a_c}$ *and* $d_a = d_{a_c}$.

4. The value of a_c satisfies

$$\text{(36)} \qquad \frac{i\bar{a}}{m(n)} = \int_{a_c}^{\bar{a}} (a - a_c)\tilde{g}_\alpha(a; n)dx + (\delta - 1)(\bar{a} - a_c)(1 - \tilde{G}_\alpha(a_c; n)).$$

5. Real money holdings $z > 0$ is given by $z = d_{a_c}$.

6. The seller-buyer ratio $n > 0$ is strictly decreasing in κ and satisfies

$$\text{(37)} \qquad m'(n)\tilde{s}_\alpha(n; \{q_a\}_{a\in A}) + m(n)\tilde{s}'_\alpha(n; \{q_a\}_{a\in A}) = \kappa.$$

7. The zero profit condition is satisfied:

$$\text{(38)} \qquad \frac{m(n)}{n} \int_{a_0}^{\bar{a}} \left[-c(q_a) + \frac{d_a}{\gamma}\right] d\tilde{G}_\alpha(a; n) = \kappa.$$

8. For any $\alpha \in (0, 1]$, the distribution of chosen goods is given by

$$\text{(39)} \qquad \tilde{G}_\alpha(a; n) = \alpha \left(\frac{e^{-n(1-G(a))} - e^{-n}}{1 - e^{-n}}\right) + (1 - \alpha)G(a).$$

Due to buyers' private information, there exists a non-empty range of utility shocks such that trade does not occur in equilibrium, i.e. $q_a = 0$. When the good chosen by a buyer within a meeting falls within this range, we call such meetings *no-trade meetings*.

Due to the liquidity constraint, if the nominal interest rate $i > 0$, there also exists a non-empty range of utility shocks such that buyers' expenditure is constrained by their money holdings, i.e. $d_a = z$. When the good chosen by a buyer within a meeting falls within this range, we call such meetings *liquidity constrained*.

The equilibrium distribution of chosen goods $\tilde{G}_\alpha$ is the same as the planner's. With probability α, the buyer faces the possibility of a one-to-many meeting. In this case, buyers always choose the highest quality seller they meet. The distribution of chosen goods therefore equals the distribution across buyers of the highest quality a among the sellers a buyer meets, conditional on meeting at least one seller. With probability $1 - \alpha$, buyers and sellers meet in random, bilateral meetings and there is no possibility of consumer choice. The distribution of chosen goods is therefore equal to the distribution of available goods. In general, for any degree of choice $\alpha \in (0, 1]$, the equilibrium distribution $\tilde{G}_\alpha$ is a weighted average of these two possibilities. In the limiting case of random matching where $\alpha \to 0$, we have $\tilde{G}_\alpha \to G$.

For any degree of choice $\alpha \in (0, 1]$, there are various possibilities for ranges of underconsumption and overconsumption relative to the first-best quantity, and there may be either under-entry or over-entry of sellers relative to the first-best allocation.

The Friedman rule ($i \to 0$) does not deliver the first-best allocation.

Corollary 1. *At the Friedman rule, competitive search equilibrium satisfies:*

1. *No-trade range. For any $a \in [a_0, a_b)$, $q_a = 0$, and $d_a = 0$.*
2. *Unconstrained range. For all $a \in [a_b, \bar{a}]$, the quantity q_a satisfies*

$$\left(a - \varepsilon_\rho(a_b; n)\frac{1 - \tilde{G}_\alpha(a; n)}{\tilde{g}_\alpha(a; n)}\right) u'(q_a) = c'(q_a) \tag{40}$$

   and the payment d_a is given by (35).

3. *No meetings are liquidity constrained: $a_c = \bar{a}$.*
4. *Parts 5-8 from Proposition 3 hold.*

For any degree of consumer choice $\alpha \in (0, 1]$, there is clearly underconsumption of all goods at the Friedman rule relative to the first best because $\varepsilon_\rho(a_b; n) > 0$. There may be either under-entry or over-entry of sellers relative to the first best. The fact that competitive search equilibrium does not deliver the first best at the Friedman rule is due to the presence of private information. As discussed later in Section 8.1, we do obtain the first-best allocation at the Friedman rule in the case of full information, i.e. when a buyer's chosen seller can observe the buyer's utility shocks prior to trade.

5.2 Effect of choice on equilibrium market power

Recall that we define firms' *market power* as firms' surplus share. Combining the equilibrium conditions (37) and (38), firms' equilibrium market power is given by

$$\underbrace{\rho_\alpha(n)}_{\text{firms' market power}} = \underbrace{\eta_m(n)}_{\text{matching elasticity}} + \underbrace{\eta_{s_\alpha}(n; \{q_a\}_{a \in A})}_{\text{surplus elasticity}}. \tag{41}$$

We highlight the dependence of the surplus elasticity on quantities here because these depend on the degree of choice α in equilibrium, unlike the first-best quantities. As we saw for the first-best level of market power, it is not clear whether firms' market power in equilibrium is increasing or decreasing in the degree of consumer choice α.

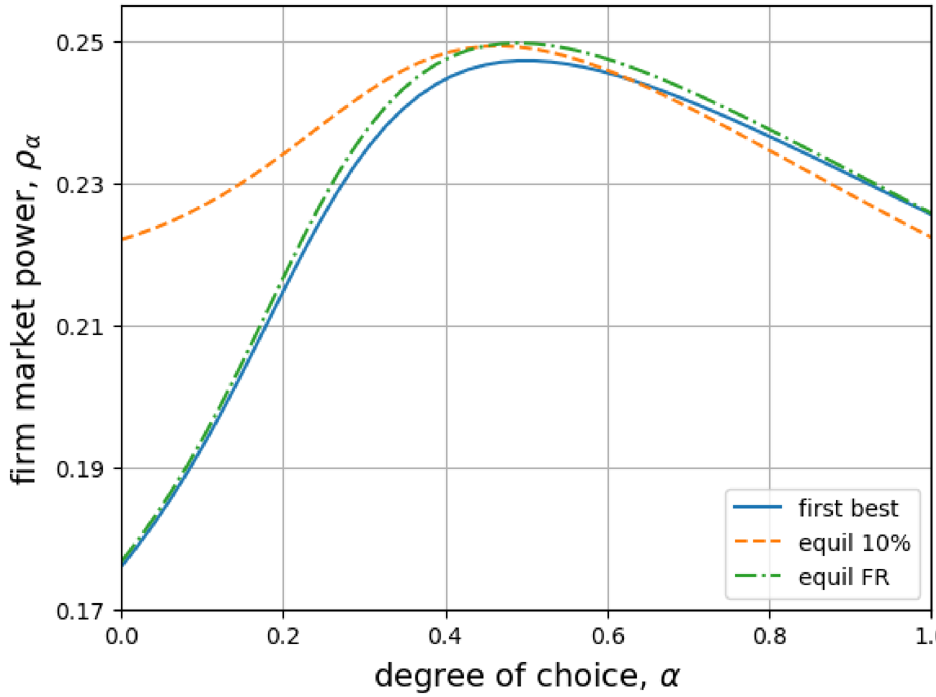

Figure 5: Firms' market power as a function of choice α at first-best and in equilibrium

Figure 5 shows the equilibrium market power of firms can vary *non-monotonically* with the degree of consumer choice α, as we saw in Section 4 for the first-best level of market power. As Figure 5 illustrates, this is true both at the Friedman rule and at an inflation rate of 10%. In both cases, firms' market power is at first increasing in the degree of consumer choice α, then it reaches a peak and is then decreasing in α.

Given that the non-monotonicity of firms' market power already appears in the first-best allocation, where the planner has complete information about buyers' utility shocks, private information cannot be the source of this non-monotonicity. At the same time, inflation cannot be the source of this non-monotonicity either, given that we observe the same non-monotonicity at the Friedman rule. The source of the non-monotonicity of firms' equilibrium market power is essentially the same as for the first-best market power, which we discussed in detail in Section 4.4.

Figure 6 provides a closer look at the way in which firms' market power varies with the degree of consumer choice. In this example, firms' equilibrium market power at the Friedman rule is always greater than it is at the first-best. However, at 10% inflation, firms' market power in equilibrium is *higher* than it is at both the Friedman rule and the first-best for low levels of consumer choice α, but *lower* than it is at both the

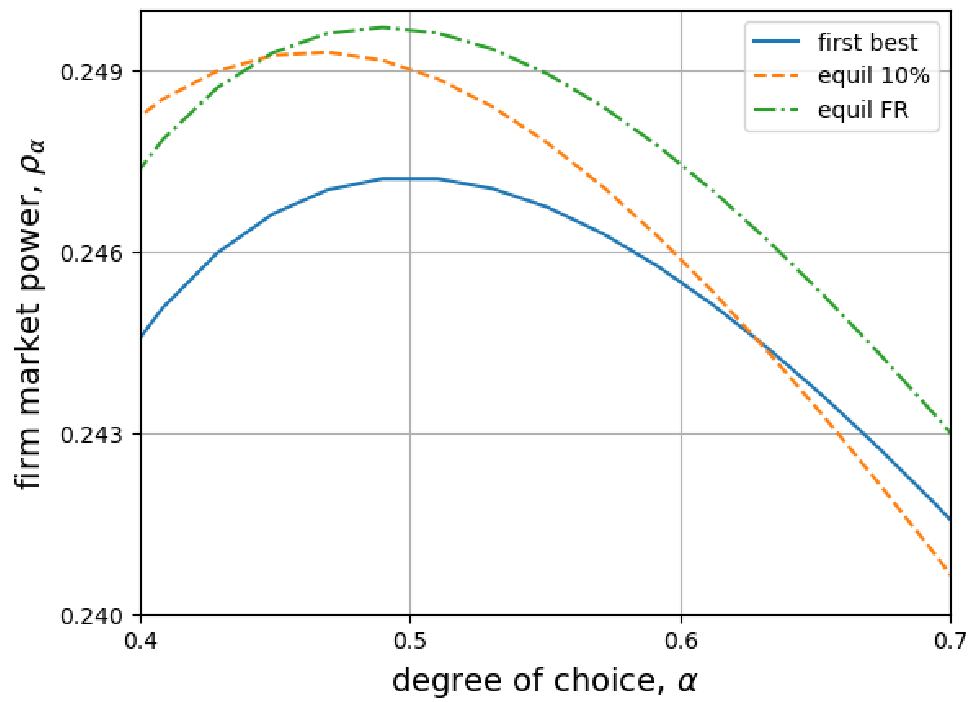

Figure 6: Firms' market power as a function of choice α at first-best and in equilibrium

Friedman rule and the first-best for high levels of α.

In this way, the effect of inflation on firms' market power appears to vary depending on the degree of consumer choice. With less consumer choice, inflation can increase firms' market power, but with more consumer choice, inflation can decrease firms' market power. Interestingly, in this example, there exists a unique level of consumer choice α (which depends on the inflation rate) such that the equilibrium and first-best levels of firms' market power are equal to each other.

Markup. An alternative measure of firms' market power is the average *markup* μ_α in the decentralized market. Defining $\tilde{d}_\alpha(n) \equiv \int_{a_0}^{\bar{a}} \frac{d_a}{\gamma} d\tilde{G}_\alpha(a;n)$, the average payment for a chosen good, and $\tilde{q}_\alpha(n) \equiv \int_{a_0}^{\bar{a}} q_a d\tilde{G}_\alpha(a;n)$, the average quantity traded, we can define the average unit price by $\tilde{p}_\alpha(n) \equiv \tilde{d}_\alpha(n)/\tilde{q}_\alpha(n)$. If we set $c(q) = q$ for simplicity, the marginal cost $c'(q)$ is always equal to one and the average markup μ_α in the decentralized market is simply equal to the average unit price, i.e. $\mu_\alpha = \tilde{p}_\alpha(n)$.

Figure 7 shows that the average markup μ_α can vary *non-monotonically* with the degree of consumer choice α. This non-monotonicity is apparent in this example both at the Friedman rule and at an inflation rate of 10%. Again, it is surprising and

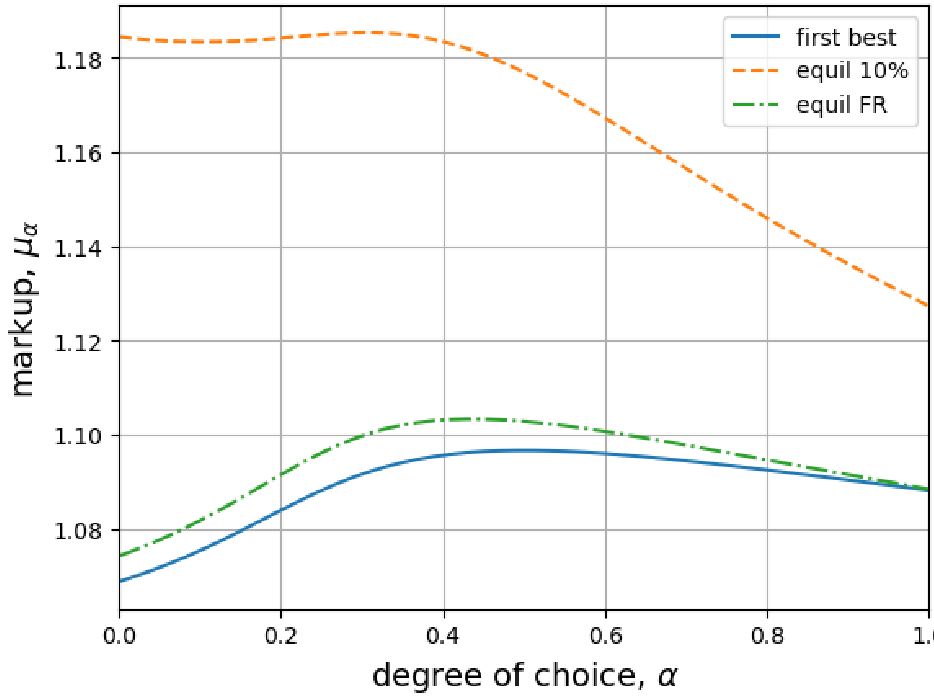

Figure 7: Markup as a function of degree of choice α at first-best and in equilibrium

counter-intuitive that a measure of firms' market power can vary non-monotonically with the degree of consumer choice. In some examples, however, we do obtain the more intuitive result that the markup is strictly decreasing in the degree of consumer choice. In fact, this is what we will observe at our baseline calibration in Section 6. This contrasts with firms' surplus share, which is always non-monotonic in α.

To understand the relationship between these two measures of market power, we can express the markup μ_α in terms of firms' surplus share $\rho_\alpha(n)$. If $c(q) = q$ then

$$\mu_\alpha = 1 + \rho_\alpha(n) \left(\frac{\tilde{s}_\alpha(n)}{\tilde{q}_\alpha(n)} \right). \tag{42}$$

The markup is clearly driven by firms' surplus share $\rho_\alpha(n)$ and the non-monotonicity in the markup is largely inherited from that of $\rho_\alpha(n)$. However, the term $\tilde{s}_\alpha(n)/\tilde{q}_\alpha(n)$ multiplying $\rho_\alpha(n)$ is itself endogenous and varies with α, which leads to a shift in the relationship between choice α and the markup. As α increases, this term tends to decrease overall because $\tilde{q}_\alpha(n)$ increases more sharply with α than does $\tilde{s}_\alpha(n)$. The higher value of this term at lower levels of α increases the markup at these levels, which partially offsets the sharp rise in $\rho_\alpha(n)$ that we see in Figure 5 at lower levels of α.

As a measure of market power, the average markup seems to be more sensitive to inflation than firms' surplus share. At higher rates of inflation, the term $\tilde{s}_\alpha(n)/\tilde{q}_\alpha(n)$ tends to increase, resulting in a higher overall markup, because $\tilde{q}_\alpha(n)$ decreases more sharply with inflation that $\tilde{s}_\alpha(n)$. This is why the markup for 10% inflation is significantly higher in Figure 7 than the markup at either the Friedman rule or the first best. By contrast to Figure 6, this is the case regardless of the level of consumer choice α.

6 Calibration

In order to quantify the effect of consumer choice on the welfare cost of inflation, we calibrate the model to match the data from Lucas and Nicolini (2015) on money demand in the U.S. from 1915-2008.[10] The period length is set to one year. We set $\beta = 1/(1+r)$ to match a real interest rate of $r = 0.03$ as in Bethune et al. (2020). We use the 3-month U.S. T-bill rate as a measure of the nominal interest rate i. The average over 1915-2008 is $i = 0.0383$. Money demand $L(i)$ is defined as $M1/GDP$.

In the model, money demand is $L(i) = z/Y$ where z is real money holdings and Y is real GDP given by $Y = x^* + m(n)\tilde{d}_\alpha(n)$, where x^* is the quantity consumed in the CM, $\tilde{d}_\alpha(n) \equiv \int_{a_0}^{\bar{a}} \frac{d_a}{\gamma} d\tilde{G}_\alpha(a;n)$, the average payment for a chosen good, and $m(n) = 1 - e^{-n}$, the probability a buyer has the opportunity to trade.

We assume that $c(q) = q$ and $u(q) = \frac{(q+\epsilon)^{1-\sigma} - \epsilon^{1-\sigma}}{1-\sigma}$ where $\sigma \in (0,1)$ and $\epsilon \approx 0$. The CM utility function is $\nu(x) = A \log x$. Since $\nu'(x^*) = 1$, we have $x^* = A$.

We assume the distribution of utility shocks has cdf $G(a) = \left(\frac{a}{\bar{a}}\right)^\psi$ on $[0, \bar{a}]$ where $\bar{a} \in \mathbb{R}_+ \backslash \{0\}$ and $\psi \geq 1$. This is a Beta distribution that satisfies Assumption 2 provided that $\psi \geq 1$.[11] To ensure that a decrease in ψ is a mean-preserving spread of G, we normalize $\bar{a} = \frac{\psi+1}{2\psi}$ so that $E_G(a) = 0.5$ regardless of ψ.[12]

For our baseline calibration, we focus on the case where $\psi = 1$. In this case, the distribution G is uniform on $[0,1]$. We focus on this case for two reasons. First, it is a standard benchmark. Second, we find that this distribution generates price dispersion that is reasonable, as we discuss below. In Section 8.2, we provide a robustness exercise where we vary the dispersion of G via the parameter ψ.

[10] Lucas and Nicolini (2015) adjust the measure of $M1$ to generate a stable money demand curve.

[11] While the assumption that $G''(a) \leq 0$ in Assumption 1 is not satisfied, we verify that $q'(a) \geq 0$ directly. The assumption that $G''(a) \leq 0$ is sufficient but not necessary for existence of equilibrium.

[12] In general, the expected value of this distribution is $E_G(x) = \psi\bar{a}/(\psi+1)$.

Parameter		*Target*	
DM utility curvature, $1-\sigma$	0.719	elasticity of money demand, η_L	-0.16
CM utility parameter, A	1.99	average money demand, $L(i)$	0.272
cost of entry, κ	0.0184	buyers' surplus share, $\theta_\alpha(n)$	0.50
degree of choice, α	0.54	decentralized market markup, μ_α	1.30

Table 1: Baseline calibration

Calibration strategy. For our baseline calibration, we calibrate four parameters $(A, \sigma, \kappa, \alpha)$ to match four targets. The target for the steady state level of money demand in the model, $L(i)$ where $i = 0.0383$, is equal to 0.272, the average money demand in the data for 1915-2008. The target for the elasticity of money demand $L(i)$ with respect to i, denoted by η_L, is equal to -0.16, the elasticity in the data for 1915-2008. Our third target is *buyers' surplus share*, which is given by $\theta_\alpha(n) = \tilde{v}_\alpha(n)/\tilde{s}_\alpha(n)$, where $\tilde{v}_\alpha(n) \equiv \int_{a_0}^{\bar{a}} v_a d\tilde{G}_\alpha(a; n)$ and $v_a \equiv au(q_a) - \frac{d_a}{\gamma}$. We target $\theta_\alpha(n) = 0.5$.[13] Observe that buyers' surplus share is equal to one minus firms' surplus share, so buyers' surplus share $\theta_\alpha(n)$ is equal to one minus firms' market power $\rho_\alpha(n)$.

Our fourth target is the *markup* in the decentralized market. Defining average quantity by $\tilde{q}_\alpha(n) \equiv \int_{a_0}^{\bar{a}} q_a d\tilde{G}_\alpha(a; n)$, the average unit price is $\tilde{p}_\alpha(n) \equiv \tilde{d}_\alpha(n)/\tilde{q}_\alpha(n)$. The DM markup is $\mu_\alpha = \tilde{p}_\alpha(n)$ since we assume $c'(q) = 1$ for our calibration. We follow Berentsen et al. (2011) in targeting $\mu_\alpha = 1.3$ to reflect a retail markup of 30%.

Discussion of calibration strategy. In the monetary search literature featuring bargaining, buyers' bargaining power is a parameter and it is generally calibrated to match either the average DM markup or the aggregate markup. Since prices are determined in competitive search equilibrium in our model, we cannot do this directly because buyers' surplus share is endogenous. Our strategy is to target buyers' surplus share at steady state through our choice of entry cost κ and match the DM markup through our choice of the parameter α. Given that we match the DM markup as a separate target anyway, the choice of target for buyers' surplus share is somewhat arbitrary, hence we simply set $\theta_\alpha(n) = 0.5$. In Section 8, we provide a robustness exercise where we vary this target and show that our main result is preserved.

Table 2 provides a summary of the equilibrium outcomes for our baseline calibration. The equilibrium features underconsumption of goods of *all* qualities (i.e. there is no

[13]Note that $\theta_\alpha(n) = \theta$ is the value of buyer's surplus share $\theta_\alpha(n)$ that would deliver the same buyer/seller shares as the Kalai (proportional) bargaining solution with parameter θ.

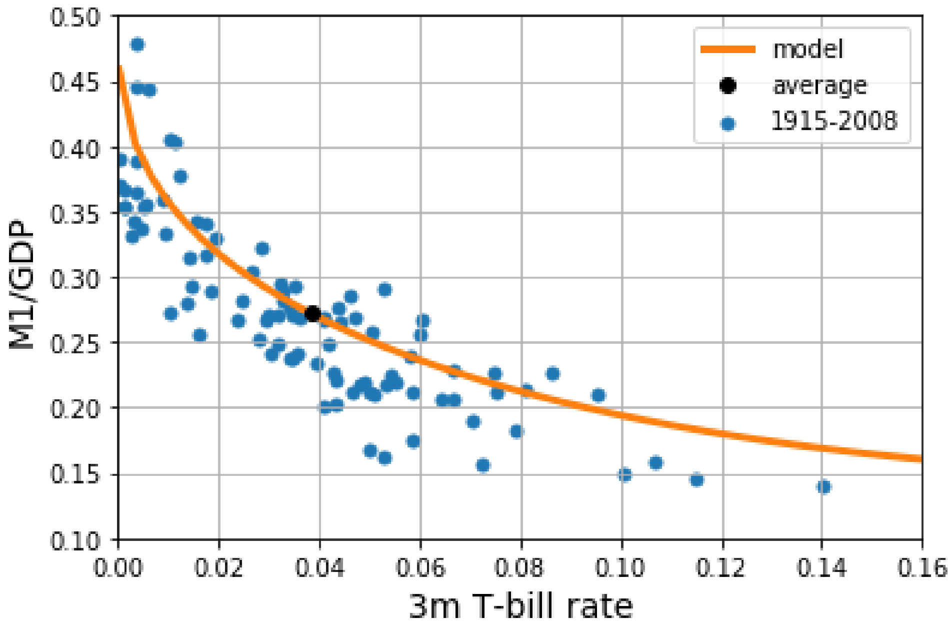

Figure 8: Data vs model predictions for money demand (by nominal interest rate i)

overconsumption). The equilibrium is also *partial trade*: around 23% of meetings do not result in any trade. Around 33% of meetings and 26% of trades are liquidity constrained. Buyers spend around 41% of their money holdings on average.

We do not target the output share of the decentralized market, but it is around 9%.[14] We also do not target price dispersion, but it is close to the empirical estimates in Kaplan and Menzio (2015). Defining unit prices by $p_a \equiv \frac{d_a/\gamma}{q_a}$ for all traded goods (i.e. DM markup since $c'(q) = 1$), *price dispersion* is defined as the standard deviation of normalized prices across all trades.[15] Price dispersion is 25% at our baseline calibration, which fits well within the range of empirical estimates, 19% to 36%, found in Table 2 of Kaplan and Menzio (2015) and is equal to their estimate of 25% for the broader definition of goods which aggregates brands (but not sizes).[16]

We provide some comparative statics results for the cost of entry κ, the inflation rate $\tau \equiv \gamma - 1$, and the degree of choice α. Table 2 summarizes the effects of a 1% increase in the parameters κ, $\gamma \equiv 1 + \tau$, and α from our baseline calibration. In

[14] In the literature, values of the DM output share vary from less than 10% in Lagos and Wright (2005) to 25% in Bethune et al. (2020) and 42% in Berentsen et al. (2011).

[15] Standard deviations are expressed as a percentage of the mean throughout the paper.

[16] We believe the "brand aggregation" measure in Kaplan and Menzio (2015) is the most relevant since goods are not strictly identical in our environment where consumers experience idiosyncratic utility or preference shocks that differ across goods.

	Baseline	$1+\tau$ ($\uparrow$ inflation)	κ ($\uparrow$ cost)	α ($\uparrow$ choice)
seller-buyer ratio, n	3.08	-2.3%	-1.1%	0.9%
meeting prob, $m(n)$	0.95	-0.4%	-0.2%	0.1%
average quality, $\tilde{a}(n)$	0.62	-0.4%	-0.2%	0.3%
average quantity, $\tilde{q}(n)$	0.20	-7.3%	-0.7%	0.9%
average payment, $\tilde{d}(n)$	0.26	-6.1%	-0.5%	0.9%
money holdings, z/γ	0.60	-8.5%	0.1%	0.3%
average surplus, $\tilde{s}(n)$	0.12	-2.6%	-0.5%	0.7%
buyer share, $\theta(n)$	0.50	-0.6%	-0.6%	-0.0%
price or markup, $\tilde{p}(n)$	1.30	1.3%	0.2%	-0.0%
price dispersion	0.25	1.1%	0.6%	-0.1%
total real output, Y	2.23	-0.7%	-0.1%	0.1%
total welfare, W	0.43	-0.5%	-0.2%	0.1%

Table 2: Equilibrium outcomes and comparative statics at baseline calibration

Table 2, we can see that greater consumer choice among buyers increases seller entry, increases the average quality of a chosen good, and increases the average quantity of goods purchased. Greater choice also increases money holdings and the average payment, as well as increasing the average trade surplus. Buyers' surplus share does not change by much when we increase the degree of choice by 1%, but it decreases slightly at the baseline calibration. The average price or DM markup also decreases slightly at baseline. Total real output and welfare (as defined below in Section 7) are both increasing in the degree of consumer choice at the baseline calibration.

Appendix D contains some figures to illustrate all of the comparative statics results over a wider range of parameter values.

7 Welfare cost of inflation

In this section, we present our estimates of the welfare cost of inflation and show how it varies with the degree of consumer choice. We start by defining total welfare in economy E by[17]

$$W(E) = m(n) \int_{a_0}^{\bar{a}} [au(q_a) - c(q_a)] d\tilde{G}_\alpha(a; n) - n\kappa + \nu(x^*) - x^* + 1. \tag{43}$$

[17] Note that adding one is a normalization that ensures $W(E)$ is positive for all calibrations we consider. It does not affect our estimates of the welfare cost of inflation.

Since consumers' utility depends on both quality and quantity, in order to calculate the consumption sacrifice in terms of quantity alone we first convert to a welfare-equivalent "representative" economy in which the quantity of goods traded is constant and quality is normalized to one. That is, we find quantity q such that

$$W(E) = m(n)[u(q) - c(q)] - n\kappa + \nu(x^*) - x^* + 1. \tag{44}$$

If total consumption is multiplied by a factor of $\Delta \in [0, 1]$, then welfare is given by

$$W(E, \Delta) = m(n)[u(\Delta q) - c(q)] - n\kappa + \nu(\Delta x^*) - x^* + 1. \tag{45}$$

We measure the welfare cost of moving from economy E to E' by the share of total consumption that consumers are willing to give up in order to go from economy E' to E. That is, the cost is $1 - \Delta$ where $\Delta \in [0, 1]$ satisfies $W(E, \Delta) = W(E')$.

We compute the welfare cost of 10% inflation relative to both 0% inflation and the Friedman rule. In particular, we find $\Delta_0 \in [0, 1]$ such that $W(\gamma = 1, \Delta_0)$ is equal to $W(\gamma = 1.1, \Delta = 1)$. The value $1 - \Delta_0$ is the percentage of total consumption that consumers are willing to give up in order to go from 10% inflation to 0% inflation. We also find $\Delta_F \in [0, 1]$ such that $W(\gamma = \beta, \Delta_F)$ is equal to $W(\gamma = 1.1, \Delta = 1)$. The value $1 - \Delta_F$ is the percentage of total consumption that consumers are willing to give up in order to go from 10% inflation to the Friedman rule.

7.1 Effect of choice on welfare cost of inflation

As we know from Proposition 2, greater consumer choice increases the level of welfare. To quantify this effect, we vary α and recalibrate the parameters (A, σ, κ) to match the first three targets of our baseline calibration. We refer to the calibration where $\alpha \to 0$ as the *random matching* calibration because all meetings are random and bilateral. We refer to the calibration where $\alpha = 1$ as the *full choice* calibration.

Starting with the random matching calibration ($\alpha \to 0$), we estimate that increasing the degree of choice to our baseline level ($\alpha = 0.54$) delivers a welfare gain worth 1.58% of total consumption. Similarly, starting at our baseline degree of choice ($\alpha = 0.54$), an increase in the degree of choice to $\alpha = 1$ delivers a welfare gain worth 2.75% of total consumption. The positive effect of greater choice on welfare is intuitive. A greater degree of consumer choice increases both the average quality of traded goods and the average quantity traded, as well as increasing seller entry and the number of trades.

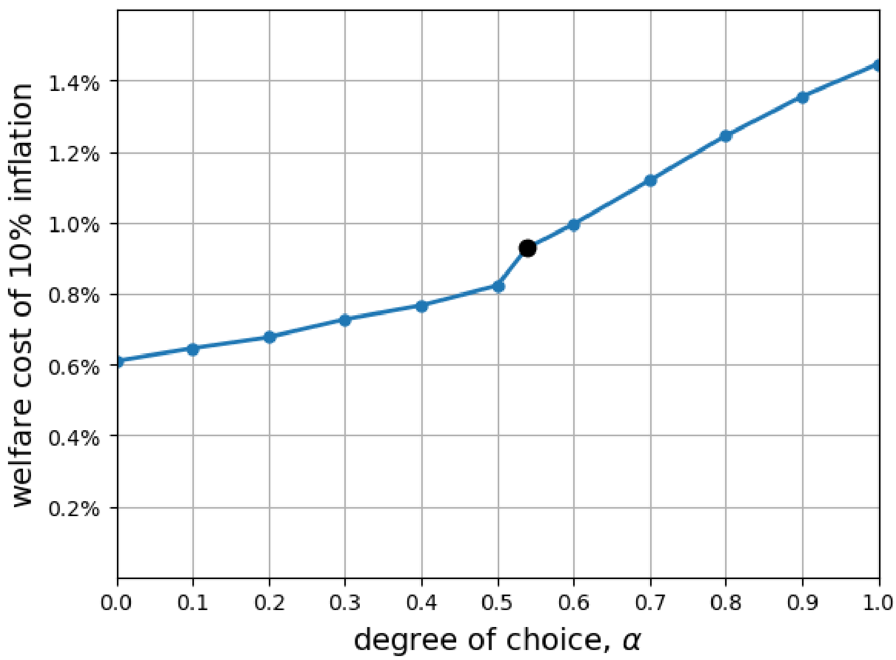

Figure 9: Welfare cost of 0% to 10% inflation for different values of α (recalibrated)

	$1-\Delta_0$	$1-\Delta_F$
Random ($\alpha = 0$)	0.61%	0.79%
Baseline ($\alpha = 0.54$)	0.93%	1.11%
Full choice ($\alpha = 1$)	1.45%	1.64%

Table 3: Welfare cost of inflation (baseline, full choice, and random matching)

The effect of consumer choice on the welfare cost of inflation is not obvious a priori. We can see the results in Figure 9, which illustrates that the welfare cost of inflation is strictly increasing in the degree of consumer choice α when we recalibrate the model (as described above) for different values of the parameter α.

Table 3 provides our estimates of the welfare cost of inflation. Recall that $1-\Delta_0$ (or $1-\Delta_F$) denotes the welfare cost of moving from 0% (or the Friedman rule) to 10% inflation. We focus on comparing our baseline calibration ($\alpha = 0.54$), full choice calibration ($\alpha = 1$), and random matching calibration ($\alpha \to 0$).[18]

At our baseline calibration ($\alpha = 0.54$), the cost of increasing inflation from 0% to 10% is 0.93% of consumption, while the cost of moving from the Friedman rule to 10% inflation is 1.11% of consumption. When we recalibrate the model after imposing

[18] Details of the full choice and random matching calibrations can be found in Appendix A.

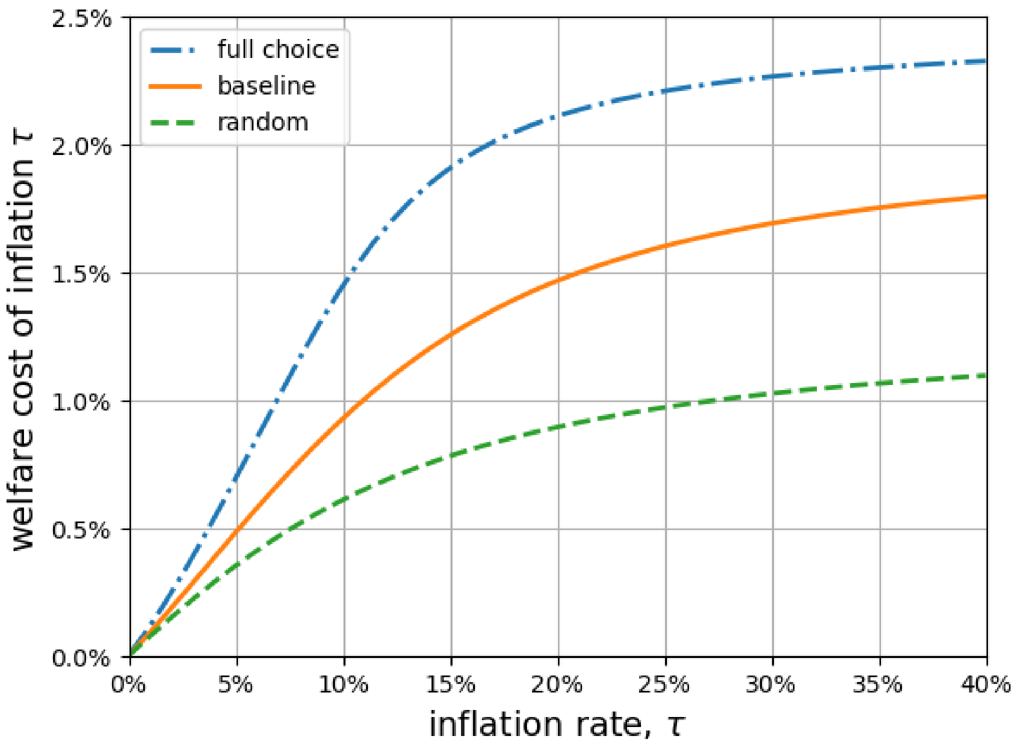

Figure 10: Welfare cost of 0% to τ inflation for different inflation rates τ

random matching ($\alpha \to 0$), the welfare cost of increasing inflation from 0% to 10% is 0.61% of consumption and the cost of moving from the Friedman rule to 10% inflation is 0.79% of consumption. On the other hand, when we recalibrate the model with full choice ($\alpha = 1$), the welfare cost of increasing inflation from 0% to 10% is more than twice as high at 1.45% of consumption, while the cost of moving from the Friedman rule to 10% inflation is 1.64% of consumption.[19]

Figure 10 depicts the welfare cost of increasing inflation from 0% to τ for various inflation rates τ at our baseline, random matching, and full choice calibrations. We can see that a welfare cost of 1% of consumption requires an inflation rate of around 11% in our baseline calibration. With random matching, a very high inflation rate of around 28% is required for the same welfare cost. With full choice, a relatively low inflation rate of around 7% delivers the same welfare cost. This suggests that economies

[19] In Appendix C, we show that our result that greater choice increases the cost of inflation still holds when we vary α and adjust κ to match the target surplus share, while keeping the utility parameters (A, σ) at their baseline levels. This confirms that the difference in the welfare cost is driven by variation in the degree of choice α, not by differences across calibrations in either the utility parameters (A, σ) or the buyer surplus share $\theta_\alpha(n)$.

	Efficient	Friedman rule	0% inflation	10% inflation
seller-buyer ratio, n	3.26	3.28	3.13	2.46
meeting prob, $m(n)$	0.96	0.96	0.96	0.91
average quality, $\tilde{a}(n)$	0.63	0.63	0.62	0.60
average quantity, $\tilde{q}(n)$	0.35	0.27	0.21	0.11
average payment, $\tilde{d}(n)$	-	0.33	0.27	0.16
money holdings, z/γ	-	1.13	0.65	0.33
average surplus, $\tilde{s}(n)$	0.14	0.13	0.12	0.09
buyer share, $\theta(n)$	-	0.51	0.50	0.46
price or markup, $\tilde{p}(n)$	-	1.23	1.28	1.46
price dispersion	-	0.24	0.25	0.27
total real output, Y	-	2.31	2.25	2.13
total welfare, W	0.45	0.44	0.44	0.42

Table 4: Equilibrium outcomes at different inflation rates (baseline calibration)

featuring a greater degree of consumer choice can experience the same level of negative welfare effects from much lower levels of inflation.

7.2 Why is the cost of inflation higher with consumer choice?

To understand better the negative effects of inflation on welfare in our model, Table 4 shows how the equilibrium outcomes change when the economy shifts from either the Friedman rule or 0% inflation to 10% inflation at the baseline calibration. We also include the first-best outcomes (given baseline $\alpha = 0.54$) for comparison.[20]

When the economy shifts from 0% to 10% inflation, the seller-buyer ratio falls by 21.4%. As a result, the meeting probability for buyers falls and average quality drops by 3.4%. Money holdings fall dramatically by 48.8%, while average quantity traded decreases by 49.2%, average payment falls by 42.3%, and average surplus drops by 24.3%. As inflation jumps from 0% to 10%, buyers' surplus share falls by 8.5%. The average price or DM markup rises by 13.7% and price dispersion rises by 9.6%. Total real output or GDP decreases by 5.2% and welfare falls by 4.5%.

The effects of inflation at our baseline calibration lie somewhere in between the effects at the two extremes of full choice ($\alpha = 1$) and random matching ($\alpha \to 0$). To see how the sensitivity of various equilibrium outcomes to changes in inflation varies with the degree of choice α, Table 5 compares the comparative statics effect of a 1% increase in the parameter $1 + \tau$ (for inflation rate τ) for our three calibrations.

[20]Notice that we have *over-entry* of sellers at the Friedman rule relative to the efficient allocation.

	Random ($\alpha = 0$)	Baseline ($\alpha = \alpha_\mu$)	Full choice ($\alpha = 1$)
seller-buyer ratio, n	-1.4%	-2.3%	-3.0%
meeting prob, $m(n)$	-0.7%	-0.4%	-0.0%
average quality, $\tilde{a}(n)$	0.0%	-0.4%	-0.5%
average quantity, $\tilde{q}(n)$	-6.1%	-7.3%	-9.2%
average payment, $\tilde{d}(n)$	-4.4%	-6.1%	-8.3%
money holdings, z/γ	-7.3%	-8.5%	-9.7%
average surplus, $\tilde{s}(n)$	-1.7%	-2.6%	-3.7%
buyer share, $\theta(n)$	-1.1%	-0.6%	-0.8%
price or markup, $\tilde{p}(n)$	1.8%	1.3%	1.0%
price dispersion	1.5%	1.1%	1.3%
total real output, Y	-0.3%	-0.7%	-1.6%
total welfare, W	-0.3%	-0.5%	-0.9%

Table 5: Effect of a 1% increase in $1 + \tau$ (inflation τ) for baseline, full choice, random

As Table 5 shows, greater choice *amplifies* the sensitivity of the economy to changes in inflation. With random matching, inflation is costly because buyers hold less money when inflation is higher, which leads to lower quantities traded and lower entry of sellers, which reduces the number of trades. When there is consumer choice, all of these effects continue to hold. However, there is an additional negative effect of inflation. With consumer choice, the *average quality* responds to changes in inflation through its effect on seller entry. In response to a 1% increase in $1 + \tau$, the average quality falls by 0.5% with full choice but is unchanged with random matching. The average quantity and money holdings are also more sensitive to changes in inflation when there is greater choice. In addition, the sensitivity of the average surplus to changes in inflation is amplified by greater choice. In response to a 1% increase in $1 + \tau$, average surplus falls by 3.7% with full choice compared to just 1.7% with random matching.

Given all these effects, which are driven by the novel effect of choice on average quality, the welfare cost of inflation is higher when there is greater consumer choice.

8 Robustness

In this section, we establish the robustness of our findings. First, we consider how our results change when we shut down the private information. Second, we examine how our results change when we relax our assumption that the distribution of utility

	Full info	Private info
Random ($\alpha = 0$)	1.55%	0.61%
Baseline ($\alpha = \alpha_\mu$)	1.69%	0.93%
Full choice ($\alpha = 1$)	2.61%	1.45%

Table 6: Welfare cost of 0% to 10% inflation for full information vs private information

shocks G is uniform and vary the degree of *dispersion* of this distribution.[21]

8.1 Effect of private information

In order to examine the effect of private information, we consider an alternative model in which the chosen seller can directly observe the buyer's utility shock for that seller prior to trade. For brevity, we refer to this as *full information.* The necessary conditions for equilibrium under full information are found in Appendix B. Both with and without choice, the Friedman rule delivers the first-best allocation.

Firms' market power. Our finding that market power can vary *non-monotonically* with the degree of consumer choice α also holds when we shut down the private information between buyers and their chosen sellers. Figure 11 illustrates that the effects of choice on market power are similar to the effects seen in Figure 5 for our model with private information. This is not surprising given that these effects are driven primarily by the non-monotonicity of firms' market power at the first-best allocation, which is the same for both full information and private information.

Welfare cost of inflation. Table 6 presents the welfare cost of inflation for the full information version of our model with both random matching and full choice, and at the baseline calibration.[22] It is clear that private information decreases the welfare cost of inflation. At the same time, private information *amplifies* the effect of consumer choice. That is, the difference in the welfare cost of inflation for different degrees of consumer choice is higher when there is private information. For example, moving from random matching to the baseline degree of choice α_μ increases the cost of inflation by only 9% with full information compared to 52% with private information.

[21]In Appendix C, we also consider how our results regarding the welfare cost of inflation change when we vary our surplus share target. We also present the results of two experiments where we shut down either endogenous seller entry or endogenous surplus shares. In all cases, we confirm our finding that the welfare cost of inflation is higher when consumers have a greater degree of consumer choice.

[22]When we shut down private information, the baseline α also varies and equals the value $\alpha_\mu = 0.36$.

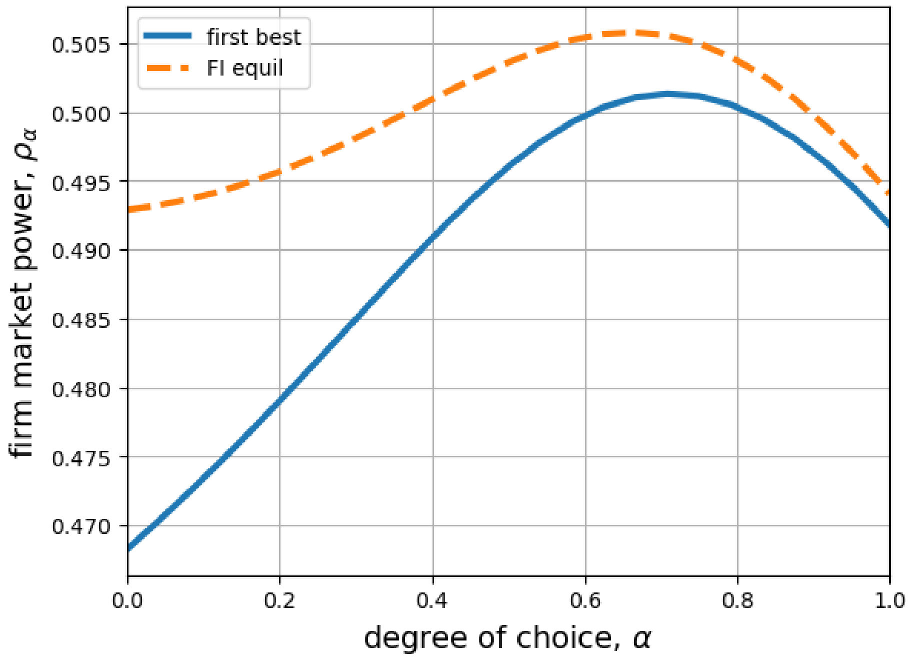

Figure 11: Firms' equilibrium market power as a function of choice α with full information

Importantly, our key result that consumer choice significantly increases the welfare cost of inflation still holds even when we shut down the private information between buyers and their chosen sellers. We focus on the private information case – instead of the full-information case – primarily because it is not realistic to assume that the buyer's chosen seller can directly observe the buyer's utility shock.

8.2 Effect of dispersion of utility shocks

We now consider the effect on our findings of a change in the *dispersion* of the distribution of utility shocks G. Recall that we assume the distribution of utility shocks has cdf $G(a) = \left(\frac{a}{\bar{a}}\right)^{\psi}$ on $[0, \bar{a}]$ where $\psi \geq 1$. To ensure that a decrease in ψ is a mean-preserving spread of G, we normalized $\bar{a}$. Therefore, an increase in ψ represents a *decrease* in the dispersion of utility shocks, without changing the mean.

Throughout the paper, we have focused on the case we use for our baseline calibration, which is $\psi = 1$, i.e. the distribution G of utility shocks is uniform on $[0, 1]$.

Firms' market power. Our finding that firms' market power $\rho_\alpha(n)$ can vary non-monotonically with the degree of consumer choice α still holds when we vary the

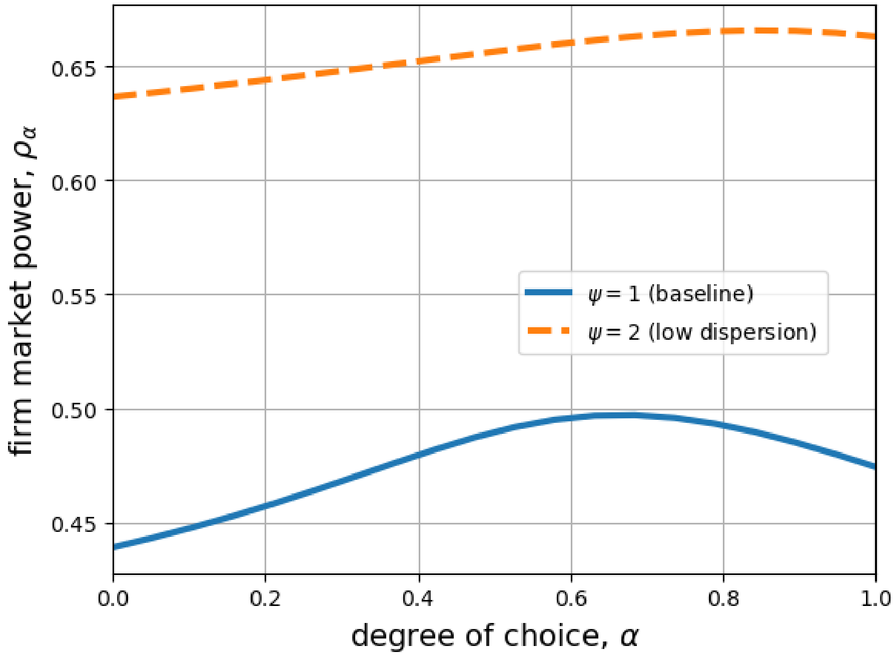

Figure 12: Firms' equilibrium market power as a function of choice α for different ψ

dispersion of the distribution of utility shocks by varying ψ. Figure 12 shows that when we decrease dispersion by increasing ψ from our baseline $\psi = 1$ to $\psi = 2$, firms' market power is still non-monotonic in choice α, although the curve is flatter. In Figure 12, we consider firms' market power at the Friedman rule, but we observe similar results both at the first-best allocation and in equilibria where there is positive inflation.

Welfare cost of inflation. Table 7 reports our estimates of the welfare cost of inflation when we vary the parameter ψ to change the dispersion of utility shocks and then recalibrate the model to match the same targets as our baseline calibration.

	$\psi = 1$	$\psi = 1.5$	$\psi = 2$
Random ($\alpha = 0$)	0.61%	0.74%	0.86%
Baseline ($\alpha = \alpha_\mu$)	0.93%	0.94%	0.97%
Full choice ($\alpha = 1$)	1.45%	1.57%	1.66%

Table 7: Welfare cost of 0% to 10% inflation for different values of parameter ψ.

While the cost of inflation is increasing in ψ (i.e. decreasing in dispersion), our key result that consumer choice increases this welfare cost is preserved when we vary ψ.

9 Conclusion

This paper considers the effects of greater consumer choice on market power and the welfare cost of inflation in an environment of retail trade featuring monetary exchange. Surprisingly, we find that firms' market power – as captured by their surplus share – can vary *non-monotonically* with the degree of consumer choice. At higher levels of choice, firms' market power may be decreasing in the degree of choice, but at lower levels of choice, greater consumer choice actually increases firms' market power. We also find that a higher degree of consumer choice makes inflation significantly more costly for an economy. This suggests that while consumers benefit overall from the ability to have more choice about their purchases, greater consumer choice may also increase firms' market power and make consumers more vulnerable to inflation.

In future work, we believe it would be interesting to use our model to explore the implications of changes in the structure of retail trade – for example, the rise of online transactions and various online platforms – for monetary policy. In particular, one area for future research would be to examine how changes in the meeting technology – which governs how consumers find and interact with sellers – may affect both firms' market power and the impact of consumer choice on the welfare cost of inflation.

Appendix A: Full choice and random matching

For the full choice calibration, we set $\alpha = 1$ and then calibrate the remaining three parameters (A, σ, κ) to match the first three targets of our baseline calibration.

Table 8 reports the calibrated parameters and targets for full choice.

Parameter		*Target*	
DM utility curvature, $1-\sigma$	0.815	elasticity of money demand, η_L	-0.16
CM utility parameter, A	1.75	average money demand, $L(i)$	0.272
cost of entry, κ	0.0081	buyers' surplus share, $\theta(n)$	0.50

Table 8: Full choice calibration ($\alpha = 1$)

Table 9 summarizes the equilibrium outcomes and the comparative statics effects of a 1% increase in parameters κ, $\gamma \equiv 1+\tau$, and α for full choice.[23]

	Baseline	$1+\tau$ ($\uparrow$ inflation)	κ ($\uparrow$ cost)	α ($\uparrow$ choice)
seller-buyer ratio, n	7.09	-3.0%	-0.9%	1.0%
meeting prob, $m(n)$	1.00	-0.0%	-0.0%	0.0%
average quality, $\tilde{a}(n)$	0.86	-0.5%	-0.1%	0.6%
average quantity, $\tilde{q}(n)$	0.36	-9.2%	-0.8%	1.8%
average payment, $\tilde{d}(n)$	0.41	-8.3%	-0.7%	1.7%
money holdings, z/γ	0.58	-9.7%	-0.1%	0.5%
average surplus, $\tilde{s}(n)$	0.11	-3.7%	-0.5%	1.4%
buyer share, $\theta(n)$	0.50	-0.8%	-0.7%	0.4%
price or markup, $\tilde{p}(n)$	1.16	1.0%	0.1%	-0.1%
price dispersion	0.12	1.3%	1.0%	-1.0%
total real output, Y	2.16	-1.6%	-0.1%	0.3%
total welfare, W	0.29	-0.9%	-0.2%	0.4%

Table 9: Comparative statics for full choice calibration ($\alpha = 1$)

For the random matching calibration, we let $\alpha \to 0$. We calibrate the remaining three parameters (A, σ, κ) to match the first three targets of our baseline calibration.

Table 10 reports the calibrated parameters and targets for random matching.

Table 11 summarizes the equilibrium outcomes and the comparative statics effects of a 1% increase in the parameters κ, $\gamma \equiv 1+\tau$, and α for random matching.

With random matching, the direction of the effects is generally the same as with full choice, but the magnitude is often significantly lower. The only differences in direction

[23] Since $\alpha = 1$ with full choice, we instead calculate the effect of a 1% *decrease* in α and then reverse the sign in Table 9.

Parameter		*Target*	
DM utility curvature, $1-\sigma$	0.641	elasticity of money demand, η_L	-0.16
CM utility parameter, A	2.06	average money demand, $L(i)$	0.272
cost of entry, κ	0.0363	buyers' surplus share, $\theta(n)$	0.50

Table 10: Random matching calibration ($\alpha \to 0$)

	Baseline	$1+\tau$ ($\uparrow$ inflation)	κ ($\uparrow$ cost)	α ($\uparrow$ choice)
seller-buyer ratio, n	1.26	-1.4%	-0.9%	1.0%
meeting prob, $m(n)$	0.72	-0.7%	-0.5%	0.5%
average quality, $\tilde{a}(n)$	0.50	0.0%	0.0%	0.2%
average quantity, $\tilde{q}(n)$	0.14	-6.1%	-0.4%	0.6%
average payment, $\tilde{d}(n)$	0.20	-4.4%	-0.1%	0.5%
money holdings, z/γ	0.60	-7.3%	0.4%	0.4%
average surplus, $\tilde{s}(n)$	0.13	-1.7%	-0.2%	0.5%
buyer share, $\theta(n)$	0.50	-1.1%	-0.7%	-0.1%
price or markup, $\tilde{p}(n)$	1.46	1.8%	0.3%	0.0%
price dispersion	0.37	1.5%	0.5%	0.0%
total real output, Y	2.21	-0.3%	0.0%	0.1%
total welfare, W	0.48	-0.3%	-0.1%	0.1%

Table 11: Comparative statics for random matching calibration ($\alpha \to 0$)

are *(i)* average quality, which does not vary in the absence of choice; and *(ii)* money holdings, which are locally decreasing in entry cost with full choice, but increasing with random matching. At our baseline calibration with partial choice ($\alpha = 0.54$), money holdings are locally increasing in entry cost, but non-monotonic (and decreasing over most of the parameter range) as Figure 14 shows.

Appendix B: Full information

With full information, there are three trading regions. In the first trading region, the IR constraint binds ($v_a = 0$) and sellers extract the full surplus, and a positive quantity $q_a > 0$ is traded. In the second trading region, both the IR constraint binds ($v_a = 0$) and the liquidity constraint (LC) binds ($d_a = z$). In the third trading region, the LC constraint binds ($d_a = z$), but buyers get positive ex post surplus ($v_a > 0$). It is straightforward to show that the Friedman rule delivers the first-best level of both entry and quantities traded with full information.

Proposition 4. *Any full information competitive search equilibrium satisfies:*

1. *IR constraint binds. For $a \in [a_0, a_r]$, $d_a/\gamma = au(q_a)$ and $q_a > 0$ solves*

$$au'(q_a) = c'(q_a).$$

2. *IR and LC constraints bind. For $a \in [a_r, a_n]$, $d_a = z$ and $q_a > 0$ solves*

$$au(q_a) = d_a/\gamma.$$

3. *LC binds. For any $a \in (a_n, \bar{a}]$, $d_a = z$ and $q_a > 0$ solves:*

$$au'(q_a) = \left(1 + \frac{i}{m(n)[1 - \tilde{G}_\alpha(a_n; n)]}\right) c'(q_a). \tag{46}$$

4. *The seller-buyer ratio $n > 0$ satisfies*

$$m'(n)\tilde{s}_\alpha(n; \{q_a\}_{a\in[a_0,\bar{a}]}) + m(n)\tilde{s}'_\alpha(n; \{q_a\}_{a\in[a_0,\bar{a}]}) = \kappa. \tag{47}$$

5. *The zero profit condition is satisfied:*

$$\frac{m(n)}{n} \int_{a_0}^{\bar{a}} \left[-c(q_a) + \frac{d_a}{\gamma}\right] d\tilde{G}_\alpha(a; n) = \kappa. \tag{48}$$

6. *The distribution of chosen goods is given by (9).*

Appendix C: Robustness of welfare cost result

In this Appendix, we describe how our results regarding the effect of consumer choice on the welfare cost of inflation vary when we change our surplus share target and when we shut down either endogenous seller entry and endogenous surplus shares.

Target surplus share. Our estimates of the welfare cost of inflation when we vary the target value of buyers' surplus share are presented in Table 12. For this exercise, we recalibrate the model using the same strategy (for baseline, full choice, and random matching). When we vary the target surplus share, note that baseline α also varies and equals the value α_μ that matches the target DM markup.

	$\theta(n) = 0.4$	$\theta(n) = 0.5$	$\theta(n) = 0.6$
Random ($\alpha = 0$)	0.25%	0.61%	0.81%
Baseline ($\alpha = \alpha_\mu$)	0.55%	0.93%	0.95%
Full choice ($\alpha = 1$)	0.90%	1.45%	1.65%

Table 12: Welfare cost of 0% to 10% inflation for different values of target buyer share

While our exact estimates of the cost of inflation depend on the target value of buyers' surplus share, it is clear from Table 12 that the cost of inflation is always increasing in the degree of choice for every level of the target surplus share.

Results of experiments. Our main result regarding the effect of choice on the welfare cost of inflation does not depend on either of two features of our model: (1) endogenous seller entry; and (2) endogenous surplus shares.

To demonstrate this, we conduct two experiments. First, we shut down endogenous seller entry by fixing the seller-buyer ratio to $n = \bar{n}$. Second, we shut down endogenous surplus shares by fixing buyers' surplus share to $\theta_\alpha(n) = \bar{\theta}$.[24] For both experiments, the equilibrium conditions are the same as Proposition 3 except that entry cost κ is replaced by endogenous J (where J is equal to equilibrium expected seller utility before entry cost). To calculate welfare, we use definition (43) and set $\kappa = 0$.

Table 13 compares the cost of inflation for our main model and the experiments. We use the same calibration strategy as our main model except we treat $\bar{\theta}$ or $\bar{n}$ as a

[24]In one sense, Experiment 2 is similar to the fixed surplus shares in a model featuring bargaining. However, it is different because we use competitive search and the equilibrium surplus shares are always the efficient ones (conditional on the quantities traded being efficient). This is why we still have relatively low costs of inflation in Experiment 2 compared to bargaining models.

	Main model	Exper 1 ($n = \bar{n}$)	Exper 2 ($\theta(n) = \bar{\theta}$)
(a) Recalibrated (A, σ)			
Random ($\alpha = 0$)	0.61%	0.58%	0.58%
Baseline ($\alpha = 0.54$)	0.93%	0.87%	1.16%
Full choice ($\alpha = 1$)	1.45%	1.22%	1.87%
(b) Baseline (A, σ)			
Random ($\alpha = 0$)	0.52%	0.56%	0.56%
Baseline ($\alpha = 0.54$)	0.93%	0.87%	1.16%
Full choice ($\alpha = 1$)	1.14%	0.96%	1.53%

Table 13: Welfare cost of 0% to 10% inflation for main model and experiments

calibrated parameter (instead of κ). For our main results (a) in Table 13, we recalibrate (A, σ) to match the money demand targets when we vary α (as for Table 3). We also include an additional check (b) where we keep (A, σ) at the baseline parameters when we vary α. For both (a) and (b), we match the surplus share target $\theta(n) = 0.5$ by adjusting κ (or $\bar{\theta}$ or $\bar{n}$) to ensure comparability of welfare costs.

With random matching, the cost of inflation is the same for both experiments because fixing $\theta_\alpha(n) = \bar{\theta}$ and fixing $n = \bar{n}$ are equivalent. This is because the standard Hosios condition applies, i.e. $\eta_m(n) = 1 - \theta_\alpha(n)$. However, when $\alpha > 0$ and there is some consumer choice, the generalized Hosios condition applies and fixing the seller-buyer ratio is *not* equivalent to fixing the surplus shares. As a result, the welfare cost of inflation differs across these two experiments when there is consumer choice.

For both experiments, Table 13 (a) shows that our main result – greater choice increases the welfare cost of inflation – is confirmed when we vary α and recalibrate only (A, σ) using the same strategy as our baseline calibration. For Experiment 1 (exogenous n), the cost of inflation is around twice as high with full choice compared to random matching.For Experiment 2 (exogenous θ), the cost of inflation is more than three times as high with full choice compared to random matching.

Table 13 (*b*) shows that, for both our main model and both of our experiments, our result that greater choice increases the cost of inflation also holds when we vary α and adjust κ to match the target surplus share, but keep the utility parameters (A, σ) equal to the baseline parameters. This confirms that, in all three cases, the difference in the welfare cost of inflation is not due to changes in the utility parameters (A, σ), or differences in the buyer surplus share $\theta_\alpha(n)$, across calibrations for different α, but is instead due solely to variation in the degree of consumer choice α.

Appendix D: Comparative statics

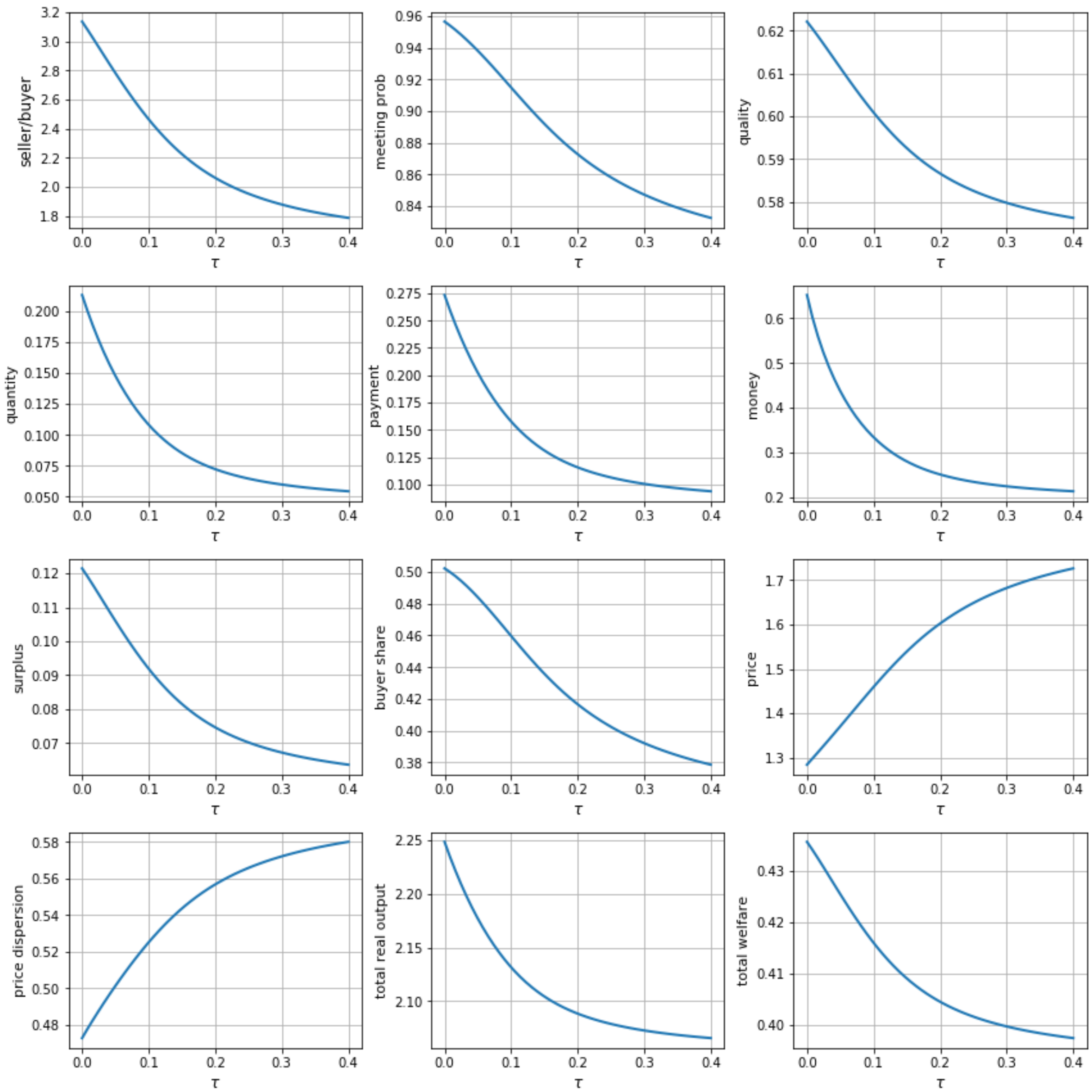

Figure 13: Comparative statics with respect to inflation rate τ

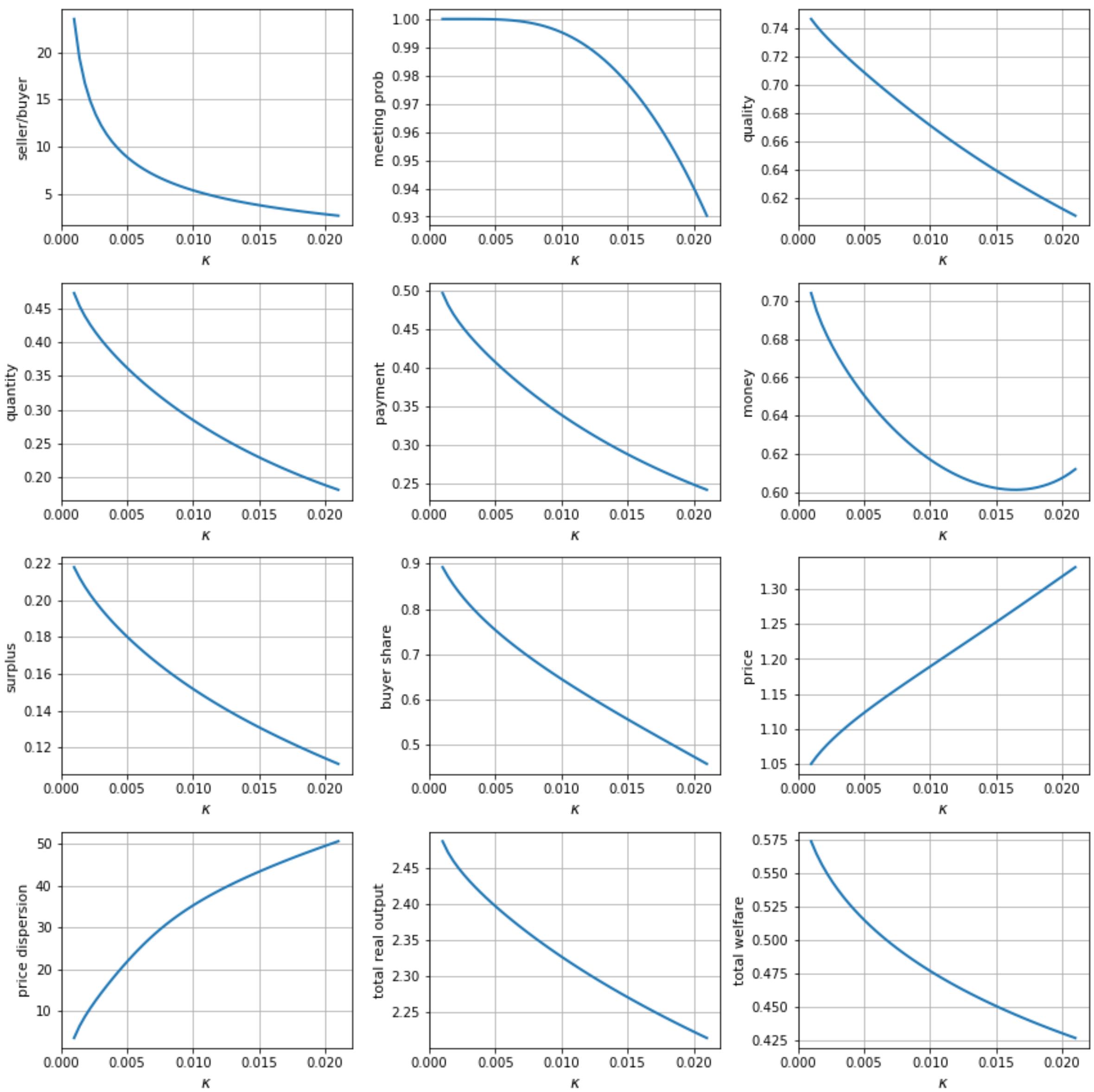

Figure 14: Comparative statics with respect to entry cost κ

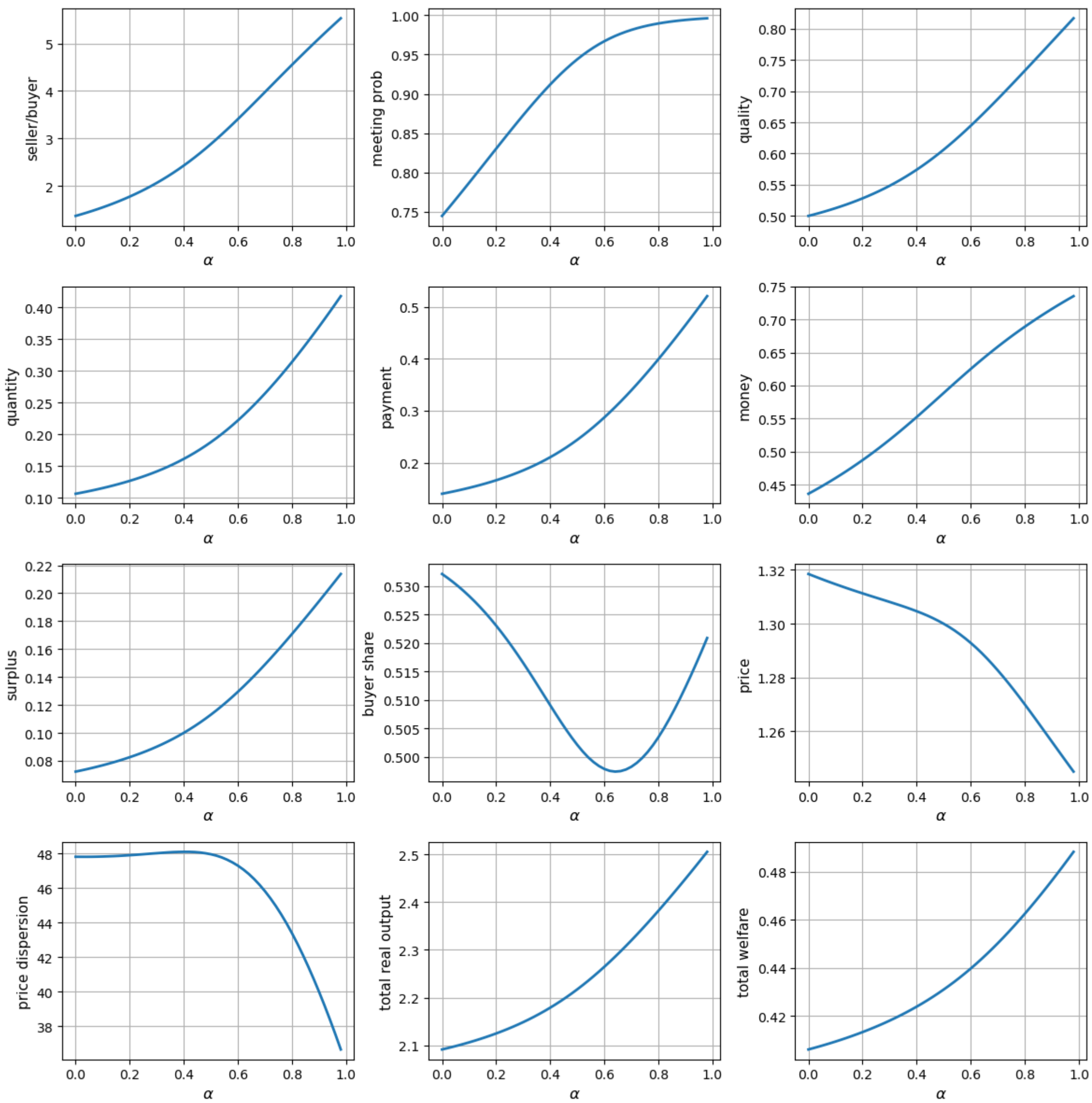

Figure 15: Comparative statics with respect to degree of choice α

Online Appendix: Proofs

Proof of Lemma 1

Since we assume the planner faces the same search frictions as the buyer, with probability $\alpha \in (0, 1]$ the planner can choose a seller in a one-to-many meeting. We verify in the proof of Proposition 1 that in this case the planner always chooses the seller with the highest utility shock among those the buyer meets. With probability $1 - \alpha$, the seller is chosen at random.

Using the fact that the distribution of the maximum of $j \geq 1$ draws is $(G(a))^j$, and weighting by the probability $P_j(n)$ that exactly j sellers meet a buyer, conditional on $j \geq 1$, we obtain

$$\tilde{G}_\alpha(a; n) = \frac{\alpha \sum_{j=1}^{\infty} P_j(n)(G(a))^j}{m(n)} + (1 - \alpha)G(a). \tag{49}$$

Given that we assume a Poisson distribution, substituting $P_j(n) = \frac{n^j e^{-n}}{j!}$ and $m(n) = 1 - e^{-n}$ into the above yields

$$\tilde{G}_\alpha(a; n) = \frac{\alpha \left(e^{-n} \sum_{j=0}^{\infty} \frac{(nG(a))^j}{j!} - e^{-n} \right)}{1 - e^{-n}} + (1 - \alpha)G(a). \tag{50}$$

Using the fact that $\sum_{j=0}^{\infty} \frac{(nG(a))^j}{n!} = e^{-n(G(a))}$, this expression simplifies to (9). ■

Proof of Lemma 2

Part 1. Taking the limit as $n \to 0$, we have

$$\lim_{n \to 0} \tilde{G}_\alpha(a; n) = \alpha \lim_{n \to 0} \left(\frac{e^{-n(1-G(a))} - e^{-n}}{1 - e^{-n}} \right) + (1 - \alpha)G(a) = G(a) \tag{51}$$

using L'Hopital's rule. Therefore, $\tilde{a}_\alpha(n) \to E_G(a)$.

Part 2. Taking the limit as $n \to \infty$, we have

$$\lim_{n \to \infty} \tilde{G}_\alpha(a; n) = \alpha \lim_{n \to \infty} \left(\frac{e^{-n(1-G(a))} - e^{-n}}{1 - e^{-n}} \right) + (1 - \alpha)G(a) = (1 - \alpha)G(a) \tag{52}$$

for any $a \in [a_0, \bar{a})$ and $\lim_{n\to\infty} \tilde{G}_\alpha(\bar{a}; n) = 1$. Therefore, $\tilde{a}_\alpha(n) \to \alpha\bar{a} + (1-\alpha)E_G(a)$.

Part 3. For $n > 0$, we have $\tilde{G}_\alpha(a; n) < G(a)$ for $a \in A$. To see this, let $w_j(n) = P_j(n)/m(n)$. Using (49), $\tilde{G}_\alpha(a; n) = \sum_{j=1}^{\infty} w_j(n)[\alpha(G(a))^j + (1-\alpha)G(a)]$. Since $\tilde{G}_\alpha(a; n)$ is a weighted average of the term $\alpha(G(a))^j + (1-\alpha)G(a)$ for all $j > 1$, and $(G(a))^j < G(a)$ for all $j > 1$ and $a \in (a_0, \bar{a})$, and $G(a)^j = G(a)$ for $a = a_0$ or $a = \bar{a}$, we have $\tilde{G}_\alpha(a; n) < G(a)$ for all $\alpha \in (0, 1]$. Therefore, $\tilde{G}_\alpha(a; n)$ first order stochastically dominates $G(a)$ and $\tilde{a}_\alpha(n) > E_G(a)$.

Part 4. Let $f : A \to \mathbb{R}_+$ such that $f' > 0$. For any n_1 and n_2 such that $n_1 > n_2$, Part 5 implies $\tilde{f}_\alpha(n_1) > \tilde{f}_\alpha(n_2)$, i.e. $\int_{a_0}^{\bar{a}} f(a)d\tilde{G}_\alpha(a; n_1) > \int_{a_0}^{\bar{a}} f(a)d\tilde{G}_\alpha(a; n_2)$. Thus $\tilde{G}_\alpha(a; n_1) \le \tilde{G}_\alpha(a; n_2)$ and $\tilde{G}_\alpha(a; n_1)$ first order stochastically dominates $\tilde{G}_\alpha(a; n_2)$.

Part 5. Applying Leibniz' integral rule gives us

$$\tilde{f}'_\alpha(n) = \int_{a_0}^{\bar{a}} f(a) \frac{\partial \tilde{g}_\alpha(a; n)}{\partial n} da. \tag{53}$$

First, we show that there exists a unique cutoff $\hat{a} \in A$ such that $\frac{\partial \tilde{g}_\alpha(a;n)}{\partial n} > 0$ for $a > \hat{a}$ and $\frac{\partial \tilde{g}_\alpha(a;n)}{\partial n} < 0$ for $a < \hat{a}$. To start with, we have

$$\tilde{g}_\alpha(a; n) = \alpha \left(\frac{ng(a)e^{-n(1-G(a))}}{1 - e^{-n}} \right) + (1-\alpha)g(a). \tag{54}$$

Differentiating (54) with respect to n, we obtain

$$\frac{\partial \tilde{g}_\alpha(a; n)}{\partial n} = \alpha g(a) \left(\frac{e^{-n(1-G(a))}[(1 - n(1 - G(a)))(1 - e^{-n}) - ne^{-n}]}{(1 - e^{-n})^2} \right) \tag{55}$$

and therefore $\frac{\partial \tilde{g}_\alpha(a;n)}{\partial n} > 0$ if and only if

$$(1 - n(1 - G(a)))(1 - e^{-n}) - ne^{-n} > 0, \tag{56}$$

or, equivalently,

$$G(a) > \frac{1}{1 - e^{-n}} - \frac{1}{n}. \tag{57}$$

Defining $\hat{a} = G^{-1}\left(\frac{1}{1-e^{-n}} - \frac{1}{n}\right)$, we have $\frac{\partial \tilde{g}_\alpha(a;n)}{\partial n} > 0$ if and only if $a > \hat{a}$.

We can use the cutoff $\hat{a}$ to rewrite $\tilde{f}'_\alpha(n)$ as follows:

$$\tilde{f}'_\alpha(n) = \int_{a_0}^{\hat{a}} f(a) \frac{\partial \tilde{g}_\alpha(a; n)}{\partial n} da + \int_{\hat{a}}^{\bar{a}} f(a) \frac{\partial \tilde{g}_\alpha(a; n)}{\partial n} da. \tag{58}$$

We therefore have $\tilde{f}'_\alpha(n) > 0$ if and only if

$$\int_{\hat{a}}^{\bar{a}} f(a)\frac{\partial \tilde{g}_\alpha(a;n)}{\partial n}da > -\int_{a_0}^{\hat{a}} f(a)\frac{\partial \tilde{g}_\alpha(a;n)}{\partial n}da > 0. \tag{59}$$

Given that $f'(a) > 0$, and both sides of (59) are positive, by definition of $\hat{a}$, a sufficient condition for $\tilde{f}'_\alpha(n) > 0$ is

$$\int_{\hat{a}}^{\bar{a}} f(\hat{a})\frac{\partial \tilde{g}_\alpha(a;n)}{\partial n}da \geq -\int_{a_0}^{\hat{a}} f(\hat{a})\frac{\partial \tilde{g}_\alpha(a;n)}{\partial n}da, \tag{60}$$

which is true if and only if $\int_{\hat{a}}^{\bar{a}} \frac{\partial \tilde{g}_\alpha(a;n)}{\partial n}da \geq -\int_{a_0}^{\hat{a}} \frac{\partial \tilde{g}_\alpha(a;n)}{\partial n}da$, or $\int_{a_0}^{\bar{a}} \frac{\partial \tilde{g}_\alpha(a;n)}{\partial n}da \geq 0$. Applying Leibniz' integral rule again, $\int_{a_0}^{\bar{a}} \frac{\partial \tilde{g}_\alpha(a;n)}{\partial n}da = \frac{\partial}{\partial n}\int_{a_0}^{\bar{a}} \tilde{g}_\alpha(a;n)da = 0$, since $\int_{a_0}^{\bar{a}} \tilde{g}_\alpha(a;n)da = 1$. Therefore, we have $\tilde{f}'_\alpha(n) > 0$. ■

Proof of Lemma 3

Parts 1 and 2. Clear.

Part 3. Suppose that $\alpha_1 > \alpha_2$. We show that $\tilde{G}_{\alpha_1}(a;n) < \tilde{G}_{\alpha_2}(a;n)$ for all $a \in A$. First, for any $\alpha \in (0,1]$, we can write

$$\tilde{G}_\alpha(a;n) = \alpha\left(\tilde{G}_1(a;n) - G(a)\right) + G(a) \tag{61}$$

where $\tilde{G}_1(a;n) = \frac{e^{-n(1-G(a))}-e^{-n}}{1-e^{-n}}$. Therefore, $\tilde{G}_{\alpha_1}(a;n) < \tilde{G}_{\alpha_2}(a;n)$ if and only if

$$\alpha_1\left(\tilde{G}_1(a;n) - G(a)\right) < \alpha_2\left(\tilde{G}_1(a;n) - G(a)\right). \tag{62}$$

We know from Lemma 2 Part 3 that $\tilde{G}_\alpha(a;n) < G(a)$ for all $\alpha \in (0,1]$ and all $a \in A$. So, in particular, we have $\tilde{G}_1(a;n) - G(a) < 0$. Therefore, the above inequality is equivalent to $\alpha_1 > \alpha_2$, which is true by assumption.

Part 4. The structure of the proof is similar to the proof of Part 5 of Lemma 2. Applying Leibniz' integral rule gives us

$$\frac{\partial \tilde{f}_\alpha(n)}{\partial \alpha} = \int_{a_0}^{\bar{a}} f(a)\frac{\partial \tilde{g}_\alpha(a;n)}{\partial \alpha}da. \tag{63}$$

First, there exists a unique cutoff $\hat{a}_\alpha \in A$ such that $\frac{\partial \tilde{g}_\alpha(a;n)}{\partial \alpha} > 0$ for $a > \hat{a}_\alpha$ and

$\frac{\partial \tilde{g}_\alpha(a;n)}{\partial \alpha} < 0$ for $a < \hat{a}_\alpha$. Starting with (54) and differentiating with respect to α,

$$\frac{\partial \tilde{g}_\alpha(a;n)}{\partial \alpha} = \tilde{g}_1(a;n) - g(a). \tag{64}$$

It can be shown that, for any given $n > 0$, there exists a unique cut-off value $a_f(n)$ such that $\tilde{g}_1(a_f(n);n) = g(a)$ and $\tilde{g}_1(a;n) > g(a)$ if and only if $a > a_f(n)$. Therefore, we have $\frac{\partial \tilde{g}_\alpha(a;n)}{\partial \alpha} > 0$ if and only if $a > a_f(n)$. Using the cutoff a_f, $\frac{\partial \tilde{f}_\alpha(n)}{\partial \alpha}$ is equal to

$$\frac{\partial \tilde{f}_\alpha(n)}{\partial \alpha} = \int_{a_0}^{a_f(n)} f(a)\frac{\partial \tilde{g}_\alpha(a;n)}{\partial \alpha}da + \int_{a_f(n)}^{\bar{a}} f(a)\frac{\partial \tilde{g}_\alpha(a;n)}{\partial \alpha}da. \tag{65}$$

We therefore have $\frac{\partial \tilde{f}_\alpha(n)}{\partial \alpha} > 0$ if and only if

$$\int_{a_f(n)}^{\bar{a}} f(a)\frac{\partial \tilde{g}_\alpha(a;n)}{\partial \alpha}da > -\int_{a_0}^{a_f(n)} f(a)\frac{\partial \tilde{g}_\alpha(a;n)}{\partial \alpha}da > 0. \tag{66}$$

Given that $f'(a) > 0$ by assumption and both sides of (66) are positive by definition of $a_f(n)$, a sufficient condition for $\frac{\partial \tilde{f}_\alpha(n)}{\partial \alpha} > 0$ is

$$\int_{a_f(n)}^{\bar{a}} f(a_f(n))\frac{\partial \tilde{g}_\alpha(a;n)}{\partial \alpha}da \geq -\int_{a_0}^{a_f(n)} f(a_f(n))\frac{\partial \tilde{g}_\alpha(a;n)}{\partial \alpha}da, \tag{67}$$

which is true if and only if $\int_{a_0}^{\bar{a}} \frac{\partial \tilde{g}_\alpha(a;n)}{\partial \alpha}da \geq 0$. Applying Leibniz' integral rule again, $\int_{a_0}^{\bar{a}} \frac{\partial \tilde{g}_\alpha(a;n)}{\partial \alpha}da = \frac{\partial}{\partial \alpha}\int_{a_0}^{\bar{a}} \tilde{g}_\alpha(a;n)da = 0$, since $\int_{a_0}^{\bar{a}} \tilde{g}_\alpha(a;n)da = 1$. So, $\frac{\tilde{f}_\alpha(n)}{\partial \alpha} > 0$. ■

Proof of Proposition 1

Taking $\alpha \in (0,1]$ as given, the first-order condition with respect to q_a is

$$m(n)[au'(q_a) - c'(q_a)]\tilde{g}_\alpha(a;n) = 0 \tag{68}$$

and the first order-condition with respect to n is

$$m'(n)\tilde{s}_\alpha(n;\{q_a\}_{a\in A}) + m(n)\tilde{s}'_\alpha(n;\{q_a\}_{a\in A}) = \kappa. \tag{69}$$

We can verify that $s_a^* = au(q_a^*) - c(q_a^*)$ is strictly increasing in a. Differentiating,

$$\frac{ds_a^*}{da} = u(q_a^*) + [au'(q_a^*) - c'(q_a^*)]\frac{dq^*}{da}. \tag{70}$$

Since $au'(q_a^*) - c'(q_a^*) = 0$ by (68) if $n^* > 0$, we have $\frac{ds_a^*}{da} = u(q_a^*) > 0$ for all $a \in (a_0, \bar{a}]$. Given that s_a^* is strictly increasing in a and $s_0^* \geq 0$, where $s_0^* \equiv a_0 u(q_0) - c(q_0)$ and $q_0 = q(a_0)$, we have $s_a^* \geq 0$ for all $a \in A$. Therefore, all chosen goods $a \in A$ are traded if $a_0 > 0$, and q_a satisfies $au'(q_a) = c'(q_a)$. If $a_0 = 0$, we have $q_a = 0$ since $\lim_{q \to 0} c'(q)/u'(q) = 0$.

Since s_a^* is strictly increasing in a, the planner chooses the seller with the highest utility shock a whenever possible, i.e. with probability α, and randomizes across sellers otherwise, i.e. with probability $1 - \alpha$. The distribution of chosen goods, $\tilde{G}_\alpha(a; n)$, is therefore equal to (9).

Existence and uniqueness of the solution to the planner's problem follows from Proposition 3. For the planner's problem, we know that $s_a^* \geq 0$ for all $a \in A$ and thus all chosen goods are traded. Setting $i = 0$ in Proposition 3 results in equilibrium conditions that are equivalent to the planner's first-order conditions. It follows that there exists a unique solution to the planner's problem with $n^* > 0$ provided that κ satisfies Assumption 4, except that $q_a^0 = q_a^*$ here since q_a^* does not depend directly on n. That is, Assumption 3 suffices.

For part 4, we show that the seller-buyer ratio n^* is strictly increasing in the degree of choice α. Define $\Lambda_\alpha(n) \equiv m(n)\tilde{s}_\alpha(n)$. The planner's first-order condition says $\Lambda'_\alpha(n^*) = \kappa$. Applying Lemma 1, we have $\tilde{s}_\alpha(n) = E_G(s_a) + \alpha[\tilde{s}_1(n) - E_G(s_a)]$ and therefore $\tilde{s}'_\alpha(n) = \alpha \tilde{s}'_1(n)$ and $\Lambda'_\alpha(n) = m'(n)\tilde{s}_\alpha(n) + \alpha m(n)\tilde{s}'_1(n)$. We know the first term is increasing in α because $\tilde{s}_\alpha(n)$ is increasing in α, and the second term is also increasing in α because $\tilde{s}'_1(n) > 0$. Therefore, $\Lambda'_\alpha(n)$ is increasing in α. In the proof of existence and uniqueness for Proposition 3, we show that $\Lambda'_\alpha(n)$ is decreasing in n. Given that $\Lambda'_\alpha(n^*) = \kappa$, we must have n^* increasing in α. ■

Assumption 4 on cost of entry κ

Assumption 4 says the expected trade surplus in the limit as $n \to 0$ must be greater than κ, otherwise no sellers enter. Since $\tilde{G}_\alpha \to G$ as $n \to 0$ by Lemma 2, $\lim_{n \to 0} \tilde{s}_\alpha(n) = E_G[au(q_a^0) - c(q_a^0)]$ where $q_a^0 \equiv \lim_{n \to 0} q_a(n)$ is given by Lemma 5.

Lemma 5. *For all $a \in [a_0, a_b]$, $q_a^0 = 0$ and, for all $a \in (a_b, \bar{a}]$, q_a^0 satisfies*

$$\left(a - \frac{1 - G(a)}{g(a)}\right) u'(q_a) = c'(q_a) \tag{71}$$

where $a_b^0 \in [a_0, \bar{a})$ is the unique solution to $\psi_G(a) = 0$.

Proof. In the limit as $n \to 0$, we have $\tilde{G}_\alpha(a;n) \to G(a)$ by Lemma 2. As $n \to 0$, we have $i/m(n) \to \infty$ so $\delta \to \infty$. Also, $1/\delta \to 0$ and $n \to 0$ implies $\phi(a;n) \to \frac{1-G(a)}{g(a)}$ on $(a_b, a_c]$. We know that $a_b = \phi(a_b;n)$, which is equivalent to $\psi_G(a_b) = 0$ where $\psi_G(a) \equiv a - \frac{1-G(a)}{g(a)}$. It follows from Assumption 2 that $\psi'_G(a) > 0$. Also, we have $\psi_G(\bar{a}) = \bar{a} > 0$ and $\psi_G(a_0) < 0$ if $a_0 g(a_0) \leq 1$, which is true because we assume $a_0 = 0$. So, there exists a unique a_b such that $\psi_G(a_b) = 0$. Finally, as $n \to 0$, the cut-off a_c solves

$$(\bar{a} - a_c)[1 - G(a_c)] = \bar{a}\left[\frac{-\psi_G(a_b)}{a_b - \psi_G(a_b)}\right](1 - G(a_b)). \tag{72}$$

Thus $\psi_G(a_b) = 0$ and $a_b > 0$, so the right side is zero, which implies $a_c = \bar{a}$. ■

Proof of Proposition 3

Our strategy is to solve for the equilibrium in two stages. First, we take z and n as given and solve for $\{(q_a, d_a)\}_{a \in A}$ (inner maximization problem). Second, we solve for z and n (outer maximization problem) given the solutions for $\{(q_a, d_a)\}_{a \in A}$. Next, we use the results to prove Parts 1 to 8 of Proposition 3.

Except for the proofs of Part 8 and Lemma 6, and the proofs of existence and uniqueness of equilibrium, which are included below, the derivations of the solution to the inner maximization problem and the outer maximization problem are lengthy and the same as those in the proof of Proposition 2 in the earlier working paper, Mangin and Bajaj (2022), so we omit these here.

For simplicity, we will sometimes suppress the dependence of $\tilde{a}_\alpha(n)$ and $\tilde{s}_\alpha(n)$ on the degree of consumer choice α and write simply $\tilde{a}(n)$ and $\tilde{s}(n)$.

Lemma 6. *For any $\alpha \in (0, 1]$, the function $q(.)$ is weakly increasing for all $a \in A$ and $q'(a) > 0$ for all $a \in (a_b, a_c)$.*

Proof. For all $a \leq a_b$, we have $q_a = 0$ and $q'(a) = 0$. For all $a \geq a_c$, we have that q_a is constant and thus $q'(a) = 0$. For $a \in (a_b, a_c)$, implicit differentiation of

$$(a - \phi(a;n))u'(q_a) = c'(q_a) \tag{73}$$

yields

$$q'(a) = \frac{-[1 - \phi'(a)]u'(q_a)}{[a - \phi(a;n)]u''(q_a) - c''(q_a)}. \tag{74}$$

Since $u'(q_a) > 0$ and $u''(q_a) < 0$ and $c''(q_a) > 0$ and $a - \phi(a;n) > 0$, we have $q'(a) \geq 0$ provided that $\phi'(a) < 1$. Next, $\phi(a;n)$ is given by

$$\phi(a;n) = \left(\frac{\delta - 1}{\delta}\right) \frac{1 - \tilde{G}_\alpha(a;n)}{\tilde{g}_\alpha(a;n)} - \frac{i}{\delta m(n) \tilde{g}_\alpha(a;n)}. \tag{75}$$

Differentiating $\phi(a;n)$ above yields

$$\phi'(a) = \frac{\delta - 1}{\delta} \left(-1 - \frac{(1 - \tilde{G}_\alpha(a;n))\tilde{g}'_\alpha(a;n)}{\tilde{g}_\alpha(a;n)^2} \right) + \frac{\tilde{g}'_\alpha(a;n)}{\tilde{g}_\alpha(a;n)^2} \frac{i}{m(n)\delta}. \tag{76}$$

It is clear that $\phi'(a) < 0$ if $\tilde{g}'_\alpha(a;n) \leq 0$, so we need only consider the case where $\tilde{g}'_\alpha(a;n) > 0$. Given that $\delta \geq 1 + \frac{i}{m(n)}$, we have $\frac{i}{m(n)\delta} \leq \frac{\delta - 1}{\delta}$, so it suffices to show

$$\frac{\delta - 1}{\delta} \left(-1 - \frac{(1 - \tilde{G}_\alpha(a;n))\tilde{g}'_\alpha(a;n)}{\tilde{g}_\alpha(a;n)^2} + \frac{\tilde{g}'_\alpha(a;n)}{\tilde{g}_\alpha(a;n)^2} \right) < 1 \tag{77}$$

or, equivalently,

$$\frac{\delta - 1}{\delta} \left(-1 + \frac{\tilde{g}'_\alpha(a;n)\tilde{G}_\alpha(a;n)}{\tilde{g}_\alpha(a;n)^2} \right) < 1. \tag{78}$$

Given that $\frac{\delta - 1}{\delta} < 1$, it suffices to show that

$$\frac{\tilde{g}'_\alpha(a;n)\tilde{G}_\alpha(a;n)}{\tilde{g}_\alpha(a;n)^2} \leq 2. \tag{79}$$

We know from the proof of Lemma 8 in Mangin and Bajaj (2022) that

$$\frac{\tilde{g}'_1(a;n)\tilde{G}_1(a;n)}{\tilde{g}_1(a;n)^2} \leq 1. \tag{80}$$

Since the above holds for any $n > 0$, it holds in the limit as $n \to 0$, so

$$\frac{\tilde{g}'_1(a;0)\tilde{G}_1(a;0)}{\tilde{g}_1(a;0)^2} \leq 1. \tag{81}$$

We have $\tilde{G}_1(a;0) = G(a)$, therefore

$$\frac{G''(a)G(a)}{g(a)^2} = \frac{\tilde{g}'_0(a;n)\tilde{G}_0(a;n)}{\tilde{g}_0(a;n)^2} \leq 1 \tag{82}$$

in the limit as $\alpha \to 0$. Thus, for both $\alpha = 1$ and $\alpha \to 0$ we have

$$\frac{\tilde{g}'_\alpha(a;n)\tilde{G}_\alpha(a;n)}{\tilde{g}_\alpha(a;n)^2} \leq 1. \tag{83}$$

For any $\alpha \in (0,1)$, letting $z = n(1 - G(a))$, we can write

$$\tilde{G}_\alpha(a;n) = \alpha \left(\frac{m'(z) - m'(n)}{m(n)} \right) + (1-\alpha)G(a) \tag{84}$$

$$\tilde{g}_\alpha(a;n) = \left(\alpha \frac{nm'(z)}{m(n)} + 1 - \alpha \right) g(a) \tag{85}$$

$$\tilde{g}'_\alpha(a;n) = \left(\alpha \frac{nm'(z)}{m(n)} + 1 - \alpha \right) G''(a) + \alpha \frac{n^2 m'(z)}{m(n)} g(a)^2 \tag{86}$$

using the fact that $m'(z) = e^{-z}$ and $-m''(z) = m'(z)$ since P_j is Poisson. Now,

$$\frac{\tilde{g}'_\alpha(a;n)\tilde{G}_\alpha(a;n)}{\tilde{g}_\alpha(a;n)^2} = \frac{\tilde{G}_\alpha(a;n)\left(\left(\alpha \frac{nm'(z)}{m(n)} + 1 - \alpha \right) G''(a) + \alpha \frac{n^2 m'(z)}{m(n)} g(a)^2 \right)}{\left(\alpha \frac{nm'(z)}{m(n)} + 1 - \alpha \right)^2 g(a)^2}. \tag{87}$$

Simplifying further, using the fact that $G'' \leq 0$ by assumption, we obtain

$$\frac{\tilde{g}'_\alpha(a;n)\tilde{G}_\alpha(a;n)}{\tilde{g}_\alpha(a;n)^2} \leq \frac{\tilde{G}_\alpha(a;n)\alpha \frac{n^2 m'(z)}{m(n)}}{\left(\alpha \frac{nm'(z)}{m(n)} + 1 - \alpha \right)^2}. \tag{88}$$

Next, expanding out the denominator and then simplifying delivers

$$\frac{\tilde{g}'_\alpha(a;n)\tilde{G}_\alpha(a;n)}{\tilde{g}_\alpha(a;n)^2} \leq \frac{\tilde{G}_\alpha(a;n)}{\alpha \frac{m'(z)}{m(n)} + 2(1-\alpha)} \tag{89}$$

and using expression (84) yields

$$\alpha \frac{m'(z)}{m(n)} = \tilde{G}_\alpha(a;n) - (1-\alpha)G(a) + \alpha \frac{m'(n)}{m(n)}. \tag{90}$$

Therefore, we obtain the following:

$$\frac{\tilde{g}'_\alpha(a;n)\tilde{G}_\alpha(a;n)}{\tilde{g}_\alpha(a;n)^2} \leq \frac{\tilde{G}_\alpha(a;n)}{\tilde{G}_\alpha(a;n) + (2 - G(a))(1-\alpha)}. \tag{91}$$

Finally, this delivers

$$\frac{\tilde{g}'_\alpha(a;n)\tilde{G}_\alpha(a;n)}{\tilde{g}_\alpha(a;n)^2} \le 1, \tag{92}$$

and thus inequality (79) holds for all $\alpha \in (0,1]$. Therefore, we have shown that $q'(a) > 0$ for all $a \in (a_b, a_c)$ for all $\alpha \in (0,1]$ provided that $G'' \le 0$. ■

Proof of Part 8 of Proposition 3

Given that v_a is increasing in a, the highest draw is always chosen by buyers whenever possible, i.e. with probability α, and buyers randomize otherwise, i.e. with probability $1-\alpha$. Therefore the cdf of chosen goods is given by (9). ■

Proof of existence and uniqueness for Proposition 3

For any $\alpha \in (0,1]$, we first prove existence and uniqueness of the solution to the inner maximization problem and then prove the same for the outer maximization problem. For the inner maximization part, fixing any $\alpha \in (0,1]$, we need to prove that, given z and n from the outer maximization problem, the solution to the inner maximization problem exists and is unique.

Inner maximization. We prove that, given z and n from the outer maximization problem, the solution to the inner maximization problem exists and is unique.

Existence. A solution to the problem exists because the set of admissible paths is non-empty and compact, and there exists an admissible path for which the objective is finite. For example, the path $q_a = 0$ and $v_a = (a-1)u(q_a)$ for all $a \in A$ is admissible (since $v_0 = 0$, $au(q_a) - v_a \le z/\gamma$, $q_a \ge 0$, $v_a \ge 0$, and $\dot{v}_a = u(q_a) + (a-1)u'(q_a)q'(a) = u(q_a)$, and $q'(a) \ge 0$). Also, the objective is finite under this path. Finally, the set of feasible paths is compact since $q_a \in [0, q^*_{\bar{a}}]$ where $q^*_{\bar{a}}$ solves $\bar{a}u'(q_{\bar{a}}) = c'(q_{\bar{a}})$ and $v_a \in [0, v_{\bar{a}}]$ where $v_{\bar{a}} = u(q^*_{\bar{a}})[\bar{a} - a_0]$ since $v_a = \int_{a_0}^{a} u(q_x)dx$.

Uniqueness. The Hamiltonian $H(q_a, v_a, \lambda_a)$ given in the proof of Proposition 2 in Mangin and Bajaj (2022), where λ_a is the co-state variable given by the Maximum Principle, is strictly concave in the control and state variables (q_a, v_a) for all a. Therefore, the solution is an optimum that solves the inner maximization problem and it is unique. To establish strict concavity, differentiating $H(q_a, v_a, \lambda_a)$ with respect to q_a

yields

$$\begin{aligned}
\frac{\partial H}{\partial q_a} &= m(n)\delta[u'(q_a) - c'(q_a)]\tilde{g}_\alpha(a;n) + \lambda_a u'(q_a), \\
\frac{\partial^2 H}{\partial q_a^2} &= m(n)\delta[u''(q_a) - c''(q_a)]\tilde{g}_\alpha(a;n) + \lambda_a u''(q_a) \equiv -X,
\end{aligned}$$

where $X > 0$, since $u''(q_a) < 0$ and $c''(q_a) > 0$. Differentiating $H(q_a, v_a, \lambda_a)$ with respect to v_a, we obtain $\frac{\partial H}{\partial v_a} = m(n)(1-\delta)\tilde{g}_\alpha(a;n)$ and $\frac{\partial^2 H}{\partial v_a^2} = 0$. Finally, $\frac{\partial^2 H}{\partial v_a \partial q_a} = 0$, so we get the Hessian matrix, $\mathbb{H} = \begin{bmatrix} -X & 0 \\ 0 & 0 \end{bmatrix}$. Since $\mathbf{x}^T \mathbb{H} \mathbf{x} < 0$ for all $\mathbf{x} \in \mathbb{R}^2 \backslash \{\mathbf{0}\}$, the Hessian $\mathbb{H}$ is negative definite and the Hamiltonian is strictly concave in (q_a, v_a).

Outer maximization. Fixing any $\alpha \in (0,1]$, we need to prove that, given $\{(q_a, v_a)\}_{a \in A}$ from the inner maximization problem, the solution (n, z) to the outer maximization problem exists and is unique, and n, z are interior solutions with $n, z > 0$ if Assumption 4 holds. To establish this result, we first prove that, for any $\alpha \in (0,1]$, there exists a non-empty set of solutions n, denoted by $N(\kappa)$, that solves the problem. We then show that equilibrium is unique if $n > 0$ for all $n \in N(\kappa)$, and finally we prove that $n > 0$ for any $n \in N(\kappa)$.

Taking $\alpha \in (0,1]$ as given, and taking $\{(q_a, v_a)\}_{a \in A}$ as given by the inner maximization problem, and ignoring constants, the outer maximization problem is

$$\max_{z,n} \left\{ m(n) \int_{a_0}^{\bar{a}} \left[a u(q_a) - \frac{d_a}{\gamma} \right] d\tilde{G}_\alpha(a;n) + (\Sigma_{a_c} - i) \frac{z}{\gamma} \right\}, \tag{93}$$

subject to $n \geq 0$ and

$$\frac{m(n)}{n} \int_{a_0}^{\bar{a}} \left[-c(q_a) + \frac{d_a}{\gamma} \right] d\tilde{G}_\alpha(a;n) \leq \kappa \tag{94}$$

with complementary slackness, where $\{(q_a, v_a)\}_{a \in A}$ solves the inner maximization.

Lemma 7. *For any $\alpha \in (0,1]$, the set $N(\kappa)$ is nonempty and upper hemicontinuous.*

Proof. Since $m(n)$ is a bijection, we can rewrite (93) in terms of m as follows:

$$\max_{z,m} \left\{ m \int_{a_0}^{\bar{a}} \left[a u(q_a) - \frac{d_a}{\gamma} \right] d\tilde{G}_\alpha(a;m) + (\Sigma_{a_c} - i) \frac{z}{\gamma} \right\}. \tag{95}$$

The objective function is continuous and, without loss of generality, we can restrict

(z, m) to the following compact set:

$$\Delta = \{(z, m) : m \in [0, 1],\ z/\gamma \in [0, \bar{a}u(q_{\bar{a}})]\} \tag{96}$$

since $q \in [0, q^*_{\bar{a}}]$ where $q^*_{\bar{a}}$ solves $\bar{a}u'(q_{\bar{a}}) = c'(q_{\bar{a}})$, and we have $z/\gamma < \bar{a}u(q_{\bar{a}})$. The constraint (94) can therefore be written as $(z, m) \in \Gamma(k)$ for all $k \geq 0$, where $\Gamma(k)$ is a continuous and compact-valued correspondence. Applying the Theorem of the Maximum (Theorem 3.6 in Stokey, Lucas, and Prescott, 1989), the correspondence that gives the set of solutions for m is nonempty and upper hemicontinuous, and therefore also $N(k)$ is nonempty and upper hemicontinuous. ■

For any $\alpha \in (0, 1]$, the following lemma establishes that any strictly positive solution $n \in N(\kappa)$ must be unique. Because $z = d_{a_c} > 0$ where $d_a/\gamma = au(q_a) - v_a$, and $\{(q_a, v_a)\}_{a \in A}$ is given by the inner maximization problem, it follows from Lemma 8 that any solution (n, z) where $n > 0$ is unique.

Lemma 8. *For any $\alpha \in (0, 1]$, if $N^+ \subseteq N(\kappa)$ and $N^+ \subseteq \mathbb{R}_+ \backslash \{0\}$, then $N^+ = \{n\}$.*

Proof. Fix $\alpha \in (0, 1]$ and consider any solution $n \in N(\kappa)$ such that $n > 0$. Defining $\Phi(n) \equiv m(n)\tilde{v}(n)$, the solutions n satisfy (37), which is equivalent to

$$m'(n)\tilde{v}(n) + m(n)\tilde{v}'(n) = 0 \tag{97}$$

or $\Phi'(n) = 0$. We show that $\Phi''(n) < 0$ and thus any solution is unique. Using (54), for any $\alpha \in (0, 1]$ we have

$$\Phi(n) = \alpha \int_{a_0}^{\bar{a}} n e^{-n(1-G(a))} v_a g(a) da + (1 - \alpha)(1 - e^{-n}) \int_{a_0}^{\bar{a}} v_a g(a) da. \tag{98}$$

Using Leibniz's integral rule, plus the envelope theorem,

$$\begin{aligned} \Phi'(n) &= \alpha \left(\int_{a_0}^{\bar{a}} e^{-n(1-G(a))} v_a g(a) da - \int_{a_0}^{\bar{a}} n(1 - G(a)) e^{-n(1-G(a))} v_a g(a) da \right) \\ &\quad + (1 - \alpha) e^{-n} \int_{a_0}^{\bar{a}} v_a g(a) da. \end{aligned} \tag{99}$$

By integration by parts on the second integral in $\Phi'(n)$ above, we obtain

$$\Phi'(n) = \alpha \left(\int_{a_0}^{\bar{a}} e^{-n(1-G(a))} (1 - G(a)) v'(a) da + e^{-n} v(a_0) \right) + (1 - \alpha) e^{-n} \int_{a_0}^{\bar{a}} v_a g(a) da > 0. \tag{100}$$

Differentiating (100), we find that

(101)

$$\Phi''(n) = -\left(\alpha \int_{a_0}^{\bar{a}} e^{-n(1-G(a))}(1-G(a))^2 v'(a)da + \alpha e^{-n} v(a_0) + (1-\alpha)e^{-n} \int_{a_0}^{\bar{a}} v_a g(a)da\right) < 0.$$

The fact that $\Phi''(n) < 0$ follows from the fact that $v'(a) = u(q_a) \geq 0$ for all a and $v'(a) > 0$ for some a and also $v(a_0) = 0$. Therefore, any solution $n > 0$ is unique. ■

From Lemma 7, we know that, for any $\alpha \in (0,1]$ and any $\kappa \geq 0$, there exists a non-empty set of solutions $N(\kappa)$ that solves problem (93). We also know that any solution z is interior, since $z/\gamma = \bar{a}u(q_{\bar{a}})$ implies $v_{\bar{a}} = \bar{a}u(q_{\bar{a}}) - \bar{z}/\gamma = 0$ and therefore $v_a = 0$ for all $a \in A$. We now prove that, for any $n \in N(\kappa)$, we have $n \in \mathbb{R}_+\backslash\{0\}$ provided Assumption 4 holds. Also, the function $n(\kappa)$ is strictly decreasing in κ.

Lemma 9. *For any $\alpha \in (0,1]$, any solution $n \in N(\kappa)$ is interior, i.e. $n \in \mathbb{R}_+\backslash\{0\}$, and the function $n(\kappa)$ is strictly decreasing in κ.*

Proof. Fix $\alpha \in (0,1]$. First, we show there exists an interior solution $n > 0$. Define $\Lambda(n) \equiv m(n)\tilde{s}(n)$. The first-order condition (37) says $\Lambda'(n) = \kappa$. We prove there exists $n > 0$ such that $\Lambda'(n) = \kappa$ if Assumption 4 holds. We have $\lim_{n\to\infty} \Lambda'(n) = 0$, and

$$\lim_{n\to 0} \Lambda'(n) = \int_{a_0}^{\bar{a}} \lim_{n\to 0} s(a; q_a(n))dG(a) \tag{102}$$

where $\lim_{n\to 0} s(a; q_a(n)) = s(a; \lim_{n\to 0} q_a(n))$. If the following condition holds:

$$E_G[au(q_a^0) - c(q_a^0)] > \kappa \tag{103}$$

where $q_a^0 \equiv \lim_{n\to 0} q_a(n)$, there exists $n > 0$ that satisfies $\Lambda'(n) = \kappa$ provided that $\Lambda''(n) < 0$ (which we prove below).

Next, any interior solution $n > 0$ is better than $n = 0$. Define the value function:

$$V(\kappa,\gamma) \equiv \max_{z,n} \left\{ m(n) \int_{a_0}^{\bar{a}} \left[au(q_a) - \frac{d_a}{\gamma} \right] d\tilde{G}_\alpha(a; n) + (\Sigma_{a_c} - i)\frac{z}{\gamma} \right\}. \tag{104}$$

Since we know that z is interior, we have $V(\kappa,\gamma) \equiv \max_n \{m(n)\tilde{v}(n)\}$ since $\int_{a_0}^{\bar{a}} \mu_a = i$. If $n = 0$ then $V(\kappa,\gamma) = 0$. If $n > 0$, $V(\kappa,\gamma) \equiv \max_n \{m(n)\tilde{s}(n) - n\kappa\}$ using constraint (94) with equality. Letting $\Lambda(n) = m(n)\tilde{s}(n)$, we have $V(\kappa,\gamma) > 0$ if $\Lambda(n) - n\kappa > 0$. Thus the candidate solution $n > 0$ is better than $n = 0$ if $\Lambda(n) > n\kappa$ for $n > 0$. Using the fact that $\Lambda'(n) = \kappa$, it suffices to show that $\Lambda''(n) < 0$ and $\frac{\Lambda'(n)n}{\Lambda(n)} < 1$ for $n > 0$.

Similarly to Lemma 8, using (54), for any $\alpha \in (0,1]$ we have

$$\Lambda(n) = \alpha \int_{a_0}^{\bar{a}} ne^{-n(1-G(a))} s(a)g(a)da + (1-\alpha)(1-e^{-n}) \int_{a_0}^{\bar{a}} s(a)g(a)da \tag{105}$$

and using Leibniz's integral rule, plus the envelope theorem, yields

$$\begin{aligned} \Lambda'(n) &= \alpha \left(\int_{a_0}^{\bar{a}} e^{-n(1-G(a))} s_a g(a)da - \int_{a_0}^{\bar{a}} n(1-G(a))e^{-n(1-G(a))} s_a g(a)da \right) \\ &\quad +(1-\alpha)e^{-n} \int_{a_0}^{\bar{a}} s_a g(a)da. \end{aligned} \tag{106}$$

Therefore, we have

$$\begin{aligned} \frac{\Lambda'(n)n}{\Lambda(n)} &= \frac{\alpha \int_{a_0}^{\bar{a}} ne^{-n(1-G(a))} s_a g(a)da + (1-\alpha)ne^{-n} \int_{a_0}^{\bar{a}} s_a g(a)da}{\alpha \int_{a_0}^{\bar{a}} ne^{-n(1-G(a))} s_a g(a)da + (1-\alpha)(1-e^{-n}) \int_{a_0}^{\bar{a}} s_a g(a)da} \\ &\quad - \frac{\alpha \int_{a_0}^{\bar{a}} n^2(1-G(a))e^{-n(1-G(a))} s_a g(a)da}{\alpha \int_{a_0}^{\bar{a}} ne^{-n(1-G(a))} s_a g(a)da + (1-\alpha)(1-e^{-n}) \int_{a_0}^{\bar{a}} s_a g(a)da}. \end{aligned} \tag{107}$$

So, $\frac{\Lambda'(n)n}{\Lambda(n)} < 1$ for $n > 0$ provided that $ne^{-n} \leq 1 - e^{-n}$, which is true.

Finally, $\Phi(n) = \Lambda(n) - n\kappa$ for $n > 0$, so $\Phi'(n) = \Lambda'(n) - \kappa$ and $\Phi''(n) = \Lambda''(n)$. Since $\Phi''(n) < 0$ from the proof of Lemma 8, we have $\Lambda''(n) < 0$. It follows that, for any $n \in N(\kappa)$, we have $n > 0$. Since we assume $\kappa > 0$, this implies $n \in \mathbb{R}_+\backslash\{0\}$.

Since n is unique by Lemma 8, there is a function $n : \mathbb{R}_+\backslash\{0\} \to \mathbb{R}_+\backslash\{0\}$ such that $n(\kappa)$ solves $\Lambda'(n) = \kappa$. Clearly, n is decreasing in κ since $\Lambda''(n) < 0$. ■

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